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Optimal ordering policies for perishable inventory with fixed shelf life

Sweta Dahiya
Assistant Professor, Department of Mathematics, Chhotu Ram Arya College, Sonipat, Haryana, India
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Abstract
Perishable inventory systems are characterized by finite product lifetimes, uncertain demand, and the economic consequences of both shortage and spoilage. These characteristics make conventional inventory policies, such as the classical economic order quantity and basic (s, Q) policies, insufficient for many practical applications. This paper develops a mathematical framework for optimal ordering policies in a periodic-review inventory system with fixed shelf life. The model incorporates stochastic demand, deterministic product expiration, ordering costs, holding costs, shortage penalties, and disposal costs. A dynamic programming formulation is presented, together with a decomposition of inventory by remaining lifetime. The resulting policy is shown to depend on the age profile of inventory rather than solely on aggregate stock. Structural properties of the optimal policy are discussed, including state dependence, order-up-to behavior, and the value of first-expiring-first-out issuing. A computational solution approach based on backward induction and value-function approximation is proposed for practical implementation. The framework provides a foundation for designing replenishment policies for food, pharmaceuticals, blood products, and other short-life products.

Keywords: Perishable inventory, fixed shelf life, dynamic programming, stochastic demand, inventory control, replenishment policy, expiration

Introduction

Perishable inventory refers to stock that loses value over time or becomes unusable after a finite lifetime. Examples include fresh food, vaccines, pharmaceuticals, blood products, and temperature-sensitive materials. In these systems, inventory decisions must balance the risk of stockouts against the cost of holding products that may expire before they are demanded.

Traditional inventory models generally assume that inventory remains usable indefinitely. Under this assumption, a unit carried from one period to the next retains the same value. Fixed-shelf-life inventory violates this assumption because the value of a unit depends on its remaining lifetime. Consequently, two inventory states with the same total quantity may have substantially different operational value if their age compositions differ.

The main objective of this paper is to formulate and analyze an optimal ordering problem for a single-item perishable inventory system with fixed shelf life. The proposed model addresses the following questions:

1. How should inventory be represented when units have different remaining lifetimes?
2. How does finite shelf life alter the optimal replenishment decision?
3. Under what conditions does an order-up-to policy remain appropriate?
4. How should holding, shortage, and disposal costs be incorporated?
5. What computational methods are suitable for solving the resulting dynamic program?

The contributions of this paper are threefold. First, it develops a finite-horizon dynamic programming model that explicitly tracks inventory by remaining lifetime. Second, it identifies structural characteristics of the optimal policy and clarifies why aggregate inventory alone is generally insufficient. Third, it presents an implementable solution procedure suitable for moderate state spaces and discusses extensions for larger systems.

Literature Review

Inventory control for perishable products has been studied using both continuous-time and discrete-time models. Early research focused on deterministic demand and fixed lifetimes, whereas later work incorporated stochastic demand, lost sales, backorders, and service-level constraints.

The classical newsvendor model determines a single-period stocking quantity by balancing overage and underage costs. Although useful for short-life products, it does not directly represent inventory carried across multiple periods. Multi-period perishable inventory models extend this framework by representing inventory according to age or remaining lifetime.

In periodic-review systems, the inventory state is commonly expressed as a vector:

$$\mathbf{x}_t = (x_{t,1}, x_{t,2}, \dots, x_{t,L}) \qquad (1)$$

Where $x_{t,\ell}$ denotes the quantity with ℓ periods of remaining life at the beginning of period t, and L is the fixed shelf life. This representation captures the fact that inventory with one period of remaining life is not equivalent to inventory with L periods of remaining life.

Several studies have shown that issuing older units before newer units is generally advantageous. This principle, known as first-expiring-first-out (FEFO), reduces avoidable disposal and preserves the flexibility of newer inventory. Other studies have analyzed base-stock and order-up-to policies for perishables, demonstrating that their optimality depends on assumptions concerning demand, lead time, disposal, and shortage treatment.

Despite these developments, practical models often simplify perishability by using an inventory-level correction or an aggregate spoilage rate. Such approximations may be inappropriate when expiration is deterministic and the shelf life is short. The present paper therefore focuses on an explicit fixed-shelf-life representation.

System Description and Assumptions

Consider a single-item inventory system operating over discrete periods $t = 1,2,\dots,T$. The following assumptions are adopted:

- Demand in period t is a nonnegative random variable D_t with probability distribution F_t.
- Each replenishment order arrives at the beginning of the period in which it is placed.
- Every unit has a deterministic shelf life of L periods.
- Demand is served using the FEFO issuing rule.
- Unmet demand is either lost or backordered, depending on the selected model variant.
- Orders are unrestricted unless an explicit capacity constraint is introduced.
- The system incurs ordering, holding, shortage, and disposal costs.
- Demand observations are available at the end of each period.
- The objective is to minimize expected total cost over the planning horizon.

The immediate-delivery assumption isolates the effect of fixed shelf life. A positive lead time can be incorporated by adding outstanding orders to the state vector.

Let $x_{t,\ell}$ represent the inventory available at the beginning of period t with ℓ periods of remaining shelf life. Thus,

$$x_{t,\ell} \geq 0, \qquad \ell = 1,2,\dots,L \tag{2}$$

The total physical inventory is

$$I_t = \sum_{\ell=1}^{L} x_{t,\ell} \tag{3}$$

However, I_t alone does not determine the future value of inventory. The complete age profile $\mathbf{x}_t$ is required.

Mathematical Model

a. Decision Variables

Let q_t denote the order quantity placed at the beginning of period t. The order is assumed to have a full shelf life of L periods and therefore enters the state as $x_{t,L}$.

The ordering decision is constrained by $q_t \geq 0$

If a maximum order capacity Q_{max} is imposed, the constraint becomes $0 \leq q_t \leq Q_{max}$

b. Demand Fulfillment

Under FEFO, demand is assigned first to the oldest available units. Define the cumulative inventory with at most ℓ periods of remaining life as

$$C_{t,\ell} = \sum_{j=1}^{\ell} x_{t,j} \tag{4}$$

For a realized demand d_t, the quantity issued from age class ℓ is

$$u_{t,\ell} = \min\left\{x_{t,\ell}, \left[d_t - \sum_{j=1}^{\ell-1} u_{t,j}\right]^+\right\} \tag{5}$$

Where $[z]^+ = \max\{z, 0\}$.

The total quantity served is

$$S_t = \sum_{\ell=1}^{L} u_{t,\ell} \tag{6}$$

For a lost-sales system, the shortage quantity is

$$B_t = (D_t - S_t)^+ \tag{7}$$

For a backorder system, the same quantity enters a backlog state rather than being lost.

c. Inventory Aging and Expiration

After demand is fulfilled, all remaining inventory ages by one period. Units with one period of remaining life expire at the end of the period. The post-demand inventory in age class ℓ is

$$y_{t,\ell} = x_{t,\ell} - u_{t,\ell} \tag{8}$$

The next-period state is therefore

$$x_{t+1,\ell} = y_{t,\ell+1}, \qquad \ell = 1,2,\dots,L-1 \tag{9}$$

And newly ordered units satisfy

$$x_{t+1,L} = q_{t+1} \tag{10}$$

The units that expire during period t are $e_t = y_{t,1}$

If disposal costs are included, the disposal cost is proportional to e_t.

d. Cost Function

Let:

- K be the fixed ordering cost;
- c be the variable purchasing cost per unit;
- h be the holding cost per unit carried to the next period;
- p be the shortage penalty per unit;
- w be the disposal cost per expired unit.

The ordering cost is

$$C_{ord}(q_t) = K\,\mathbb{I}(q_t > 0) + cq_t \tag{11}$$

Where $\mathbb{I}(\cdot)$ is the indicator function.

The expected one-period cost is

$$C_t(\mathbf{x}_t, q_t, D_t) = K\,\mathbb{I}(q_t > 0) + cq_t + h\sum_{\ell=2}^{L} y_{t,\ell} + pB_t + we_t \tag{12}$$

If holding costs differ by remaining lifetime, h can be replaced by h_ℓ. Similarly, expiration may generate a salvage value rather than a disposal cost. In that case, the term we_t is replaced by $-se_t$, where s is the salvage value per unit.

Dynamic Programming Formulation

a. Value Function

Let $V_t(\mathbf{x})$ denote the minimum expected cost from period t through period T, given beginning-of-period inventory state $\mathbf{x}$. The terminal value is

$$V_{T+1}(\mathbf{x}) = 0$$

Unless a terminal inventory, disposal, or service requirement is specified.

The Bellman equation is

$$V_t(\mathbf{x}) = \min_{q \geq 0} \mathbb{E}_{D_t}[C_t(\mathbf{x}, q, D_t) + V_{t+1}(\Phi(\mathbf{x}, q, D_t))] \qquad (13)$$

where $\Phi(\mathbf{x}, q, D_t)$ is the state-transition function induced by demand fulfillment, expiration, and aging.

The optimal ordering policy is

$$q_t^*(\mathbf{x}) \in \operatorname{argmin}_{q \geq 0} \mathbb{E}_{D_t}[C_t(\mathbf{x}, q, D_t) + V_{t+1}(\Phi(\mathbf{x}, q, D_t))] \qquad (14)$$

This equation shows that the optimal order quantity depends on the complete remaining-life profile rather than only on total inventory.

b. Finite-Horizon Objective

The initial-state optimization problem is

$$\min_{\pi} \mathbb{E}^{\pi}[\textstyle\sum_{t=1}^{T} C_t\,(\mathbf{x}_t, q_t, D_t)] \qquad (15)$$

Where π denotes an admissible ordering policy.

For a policy π, the decision satisfies

$$q_t = \pi_t(\mathbf{x}_t) \qquad (16)$$

The optimal policy is consequently

$$\pi^* = \{\pi_1^*, \pi_2^*, \ldots, \pi_T^*\} \qquad (17)$$

Structural Properties of the Optimal Policy

a. Dependence on Inventory Age

Two states may have equal total inventory but different optimal order quantities. For example, consider states $\mathbf{x}$ and $\mathbf{x}'$ satisfying

$$\textstyle\sum_{\ell=1}^{L} x_\ell = \sum_{\ell=1}^{L} x_{\ell'} \qquad (18)$$

While $x_1 > x_{1'}$

The first state contains more inventory that is close to expiration. It may therefore justify a smaller order, even though aggregate inventory is identical.

This property distinguishes perishable inventory control from nonperishable inventory control. The state variable must preserve information about remaining lifetime.

b. FEFO Issuing

Under nonnegative disposal costs and no quality advantage for newer units, FEFO is weakly optimal. Issuing an older unit before a newer unit cannot reduce current service and preserves the newer unit for future periods. Consequently, FEFO reduces the probability that usable inventory expires unused.

c. State-Dependent Order-Up-To Behavior

For nonperishable inventory, the optimal policy often has a simple order-up-to form. For perishable inventory, a related structure may exist, but the target depends on the age profile:

$$q_t^*(\mathbf{x}) = [S_t(\mathbf{x}) - A_t(\mathbf{x})]^+ \qquad (19)$$

Where:

- $S_t(\mathbf{x})$ is a state-dependent target position;
- $A_t(\mathbf{x})$ Is the effective available inventory after accounting for age and expected expiration?

The target is generally not constant because the marginal value of inventory varies with remaining shelf life.

d. Effect of Disposal Cost

An increase in disposal cost generally discourages large orders, especially when demand uncertainty is high and shelf life is short. The effect is strongest for units whose expected demand coverage extends beyond their usable lifetime.

e. Effect of Shortage Cost

An increase in shortage cost generally increases safety stock and order quantities. However, the response is moderated by shelf life. Holding additional stock is less attractive when the probability of expiration is high.

f. Effect of Shelf Life

As L increases, the system approaches a nonperishable inventory model, provided expiration costs and quality deterioration become negligible. As L decreases, replenishment decisions become more frequent and more sensitive to short-term demand forecasts.

Special Cases

a. One-Period Shelf Life

When $L = 1$, all inventory expires after the current period. The problem reduces to a sequence of independent single-period stocking decisions. With no fixed ordering cost, the optimal quantity is characterized by the critical-fractile condition

$$F_t(q_t^*) = \frac{p-c}{p+w} \qquad (20)$$

When the costs are defined so that c is the purchase cost, p is the shortage cost, and w is the disposal cost. The exact expression changes if holding, salvage, or sales revenue is included in the objective.

b. Shelf Life

As $L \to \infty$ and expiration costs vanish, the age profile becomes irrelevant. The state can be reduced to aggregate inventory, yielding a conventional stochastic inventory model.

c. Deterministic Demand

If demand is deterministic, the dynamic program becomes a deterministic optimization problem. In this case, orders can often be scheduled to avoid expiration completely, provided the initial inventory and replenishment flexibility are sufficient.

d. No Disposal Cost

If expired units have no disposal cost, expiration still creates an economic loss because purchased units fail to generate sales and may incur holding costs. The model remains perishable even when $w = 0$.

Solution Methodology

a. Exact Dynamic Programming

For discrete demand and bounded inventory, exact dynamic programming can be implemented by enumerating all feasible states and actions. The procedure is:

1. Define the finite state space of age-profile vectors.
2. Initialize the terminal value function.
3. For $t = T, T-1, \dots, 1$, evaluate the expected cost of every feasible order quantity.
4. Select the minimizing order quantity for each state.
5. Store the value and policy functions.

The computational complexity increases rapidly with both shelf life and maximum inventory. If each age class can contain M units, the number of age-profile states is approximately

$(M+1)^L$

This state-space growth is commonly referred to as the curse of dimensionality.

b. Value-Function Approximation

For larger systems, the value function can be approximated using:

- state aggregation;
- linear or piecewise-linear approximation;
- rollout policies;
- approximate dynamic programming;
- reinforcement learning;
- Policy-function approximation.

A useful state aggregation retains total inventory together with an expiration-risk measure, such as

$$R_t = \sum_{\ell=1}^{L} \omega_\ell x_{t,\ell}$$

Where ω_ℓ assigns greater weight to inventory with shorter remaining life.

Although aggregation reduces computational effort, it may lose information needed to distinguish states with different age distributions. Therefore, the approximation should be evaluated against an exact model on smaller instances.

c. Simulation-Based Evaluation

A candidate policy should be tested under multiple demand distributions. Important performance measures include:

- expected total cost;
- fill rate;
- probability of stockout;
- average expired quantity;
- average inventory;
- order frequency;
- waste-to-demand ratio.

For a simulation horizon of T periods, the waste ratio may be defined as provided the denominator is positive.

$$\text{Waste Ratio} = \frac{\sum_{t=1}^{T} e_t}{\sum_{t=1}^{T} D_t} \quad (21)$$

Comparisons should include a nonperishable-policy benchmark, a myopic policy, and the proposed age-dependent policy.

Managerial Implications

The model yields several practical implications.

First, inventory reports should distinguish usable stock from total physical stock. A warehouse containing a large quantity of near-expiry inventory may require no immediate replenishment, whereas the same total quantity in newly received stock may justify a different decision.

Second, FEFO issuing should be integrated with replenishment planning. An ordering policy designed under FEFO assumptions may perform poorly if warehouse operations instead use first-in-first-out or arbitrary picking.

Third, demand forecasts should be aligned with product lifetime. Long-horizon forecasts are less valuable for products with very short shelf lives because inventory ordered today may not remain usable when the forecasted demand occurs.

Fourth, disposal cost should include both direct and indirect components. Direct disposal expenses may be small, while reputational damage, regulatory requirements, reverse logistics, and environmental effects may be significant.

Finally, a single service-level target may be insufficient. For perishable inventory, managers should jointly monitor service performance and waste. A policy that achieves a high fill rate by carrying excessive stock may be economically and environmentally inefficient.

Extensions

The proposed model can be extended in several directions.

a. Positive Lead Time

If replenishment requires τ periods, outstanding orders must be included in the state:

$$s_t = \left(x_t, o_{t,1}, o_{t,2}, \dots, o_{t,\tau}\right)$$

Where $o_{t,j}$ is the quantity scheduled to arrive in j periods?

b. Random Shelf Life

The deterministic lifetime assumption can be replaced by a random lifetime distribution. In that case, the transition function must account for probabilistic deterioration and expiration.

c. Multiple Products

For multiple products, shared storage capacity and joint ordering costs create coupling constraints. A capacity constraint may be written as

$$\sum_{i=1}^{N} v_i \sum_{\ell=1}^{L_i} x_{i,t,\ell} \le C$$

Where v_i is the storage volume per unit of product i and C is the available capacity.

d. Substitution and Product Upgrading

In pharmaceutical and food systems, products may be substitutable. A product with longer remaining life may serve demand intended for a product with shorter remaining life, subject to quality or regulatory restrictions.

e. Carbon and Sustainability Costs

Environmental costs can be incorporated by adding a waste-related penalty:

$$C_{environment,t} = \gamma e_t$$

Where γ represents the environmental cost per discarded unit. This formulation enables simultaneous evaluation of financial and sustainability objectives.

Conclusion

This paper formulated an optimal ordering problem for perishable inventory with fixed shelf life. The central modeling result is that the inventory state must preserve the quantity in each remaining-life category. Aggregate inventory is generally insufficient because units with different expiration dates have different economic values.

A dynamic programming formulation was developed to balance purchasing, holding, shortage, and disposal costs. The analysis demonstrated that the optimal ordering policy is state dependent, that FEFO issuing is generally desirable, and that shelf life strongly influences the trade-off between service and waste. Exact dynamic programming provides an appropriate solution method for small and moderate systems, while approximate dynamic programming and simulation-based methods are suitable for larger applications.

Future research should investigate positive lead times, random shelf life, demand learning, multiple products, capacity constraints, and sustainability objectives. These extensions are particularly relevant for modern supply chains in which waste reduction and service reliability are equally important.

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