International Journal of Research in Advanced Engineering and Technology
www.allengineeringjournal.in
ISSN: 2455-0876
Received: 07-03-2025, Accepted: 08-04-2025, Published: 23-04-2025
Volume 11, Issue 1, 2025, Page No. 30-37

Stress wave dynamics in drill strings: Influence of axial forces from drilling jars

Hasan N Al-Mamoori, Haifeng Ma, Jialin Tian
Department of Mechanical, Engineering in Southwest Petroleum University, Gas Equipment Technology Sharing and Service Platform of Sichuan Province, School of Mechanical Engineering, Southwest Petroleum University, Chengdu, Sichuan, China
Corresponding Author: Hasan Nihad

Abstract

This research presents a comprehensive mathematical model for wave propagation within a drill string subjected to external axial forces applied by a drill jar. The study examines how pulse characteristics, including displacement, velocity, and energy distribution, evolve and vary at various points along the drill string. A key focus is comparing the impact point with another critical location further along the drill string, especially in areas prone to sticking. The analysis examines the dynamic transfer of energy within the drill string under natural damping conditions using MATLAB-based simulations. These simulations are particularly valuable for highlighting how energy is transferred, dissipated, and absorbed at sticking points, providing insights into the resulting stress and strain patterns. The study highlights the impact of natural damping on wave behavior, illustrating how it attenuates specific high-frequency components while promoting energy retention over extended durations. This clarifies how drill strings can be optimized to reduce stress concentrations and avoid potential failure points. Additionally, the results illuminate the behavior of stress waves as they interact with different sections of the drill string, indicating zones where structural weaknesses are likely to emerge. These findings contribute to a broader understanding of wave dynamics in drilling operations and inform strategies for improving drilling performance, minimizing mechanical wear, and enhancing overall operational safety. By advancing predictive capabilities for wave behavior in drill strings, this research supports the design of more resilient drilling systems that are better equipped to handle challenging downhole environments and deep drilling conditions.

Keywords: Drill String, Wave Propagation, Time-Varying Axial Force, Displacement Analysis, Cumulative Work, MATLAB Simulation.

Introduction

Wave propagation in drill strings subjected to downhole axial forces (external, e.g., from drilling jar impacts) is critical for optimal performance and safety during drilling operations. Drill jars are essential tools used to initiate high amplitude shock waves within the drill string, jarring stuck pipe loose and generation pressure pulses along the system, which impacts both vibrational responses of the complete structure clear through a fully strung riser (drill pipe displacement-velocity-stress also communicated at locations where sticking is expected) [1]. Another example is elastic bars, which are essential mechanical elements for various applications, and their tensile or compressive loading allows wave propagation. Deciphering the role of dynamic and quasi-static loadings on structural stability can be traced back to this behavior, whereby it is controlled by theoretical frameworks that account for shedding trajectories as solutions in a seven-dimensional phase space related to axial wave propagation over length scales where damping prevails. This framework is essential at the atomic and larger scales and produces valuable insights, which can also aid in studying drill string dynamics under axial forces. This work contributes to increased operational efficiency with greater structural integrity [2].

These challenges become even more acute as drilling operations progress to deeper depths; environmental factors such as high temperatures and pressure variations affect wave propagation. In this environment, traditional mud pulse telemetry systems, with their inherent signal attenuation, face severe restrictions that hinder deep and complex drilling conditions [3]. There is also the case that the interface of drilling fluids with formation boundaries can create additional complications in wave propagation. The drilling fluid increases damping and viscous forces, which inhibit the wave amplitude from decreasing and frequency shifts signals, causing real-time data reliability degradation. These interactions can deplete and scatter the waves, complicating signal analysis efforts and presenting significant barriers to accurate data interpretation. This may also introduce time delays in wave transmission, giving the drilling system lower responsiveness to changes. This type of interaction implies a new and profound analysis of the drilling fluid behavior upon geological formations (boundaries) and other conditioned rock. Improving wave and vibration forward models for various drilling conditions can lead to better drilling efficiencies and help reduce operational challenges [4, 5].

Axial forces complicate the response of drill strings to vibrations and may result in complex or multifaceted vibrational patterns given specified mild inclinations for horizontal sections and straight-hole scenarios. In these cases, my vibration can significantly affect frictional forces between the drill string and borehole walls, which may drive higher energy requirements and create difficulty in steering. In addition, these axial forces are combined with various other dynamic loads that frequently amplify vibrational behavior – afflicting stability and efficiency during the drilling process [2]. Sarıbay has examined the stress wave phenomena due to impact-type dynamic loading on a pre-cracked bar subjected to an axial sudden load and concluded that these waves are amplified intensively around possible crack initiation points, alluding that energy density fluctuations play significant roles in determining both intensities of Dynamic Loading Wave (DLW) source as well as propagation nature. This research also highlights the importance of considering regions subject to more localized stress concentrations, as they can be particularly susceptible to receiving increased wave energy, ultimately leading to material fatigue and increasing their vulnerability over time to structural damage [6].

Physically, the balance of these forces is responsible for keeping the drill string stable; herein, they state that axially impact loads and transient hydraulic loading come to a complex profile in a torque-drag relationship. Such profiles have a significant impact on overall operational efficiency by influencing energy demands and the wear of drill strings. Therefore, the importance of understanding how both forces interact lies in their primary responsibility for ensuring control over a drill string's dynamic response and stability through integration with other technical issues during drilling. [7].

Symbolic computation methods were particularly helpful in understanding the low-frequency longitudinal impact wave, which arises from mechanical expansion induced by axial loading, and numerous applications were identified.

Methodology: In a series of articles over the last few years, Keer and Hou demonstrated that these procedures enhance the accuracy of mathematical models in simulating wave propagation through concrete, providing greater insight into how axial forces affect waves in the studied samples. Symbolic computation enables researchers to examine the nature of wave propagation with greater accuracy, and this is key in anticipating dynamic responses within the drill string itself, allowing design alterations or operational plans to be fine-tuned as needed. [8]. Hu and Eberhard improved this modeling by using symbolic computation on the dynamic equations of stress waves, which enhanced their accuracy in predicting these models under complex axial loading conditions. Symbolic computation demonstrates how the accuracy of stress wave analysis in environments with highly varying axial forces, which can impact material integrity, may be enhanced. This method offers a clear insight into the stress distribution along the drill string for more precise evaluations of wave behavior, which in return allows detailed prediction of possible weak points occurring during operations [9].

Drumheller noted that the well-known periodic nature of drill strings, composed by merely coupling short sections of pipes utilizing threaded joints, for example, can introduce dispersive wave propagation due to this interaction. At increasing depths, the effect becomes more significant as the periodic structure of a drill string meets various environmental factors and forms a highly complex transmission medium. These dispersive properties contribute to wave behavior, resulting in increasingly complex dynamic responses of the drill string with increasing depth, and significantly increase the difficulty of predicting or controlling the system when external conditions significantly enhance signal transmission through soils or wave propagation [10].

Additional studies on the propagation of drill string pulse energy have emphasized that modeling methods used must accurately depict stress distribution and deformation evolution. One study, for example (Drumheller), showed that although acoustic carrier waves may have the potential to increase transmission rates, actual modeling is necessary due to dispersion terms included by the structural periodicity of drill strings in order be able to predict wave behavior in realistic drilling conditions properly [11]. Measurements of stress-wave transmission in fluid-filled drill strings have also demonstrated that wave propagation properties, such as velocity and amplitude, can drastically influence solid particles floating in drilling mud waters. The presence of these particles contributes to wave attenuation and higher energy dissipation by changing the overall transmission properties of stress waves. The particulate content of the fluids will also interact with the structural properties of a drill string to alter stress waves traveling through these more complicated dynamic environments, which can affect how efficiently drilling happens and possibly degrade equipment life [5], [12].

This paper is aimed at presenting a wave propagation model of the drill string subjected to axial forces from drilling jars for studying pulse behavior in time over regions along the length of string using mathematics. The study examines displacements, velocities, and energy transfer at various load cases, all under natural damping due to the drill string materials available to the operator, with a focus on stress distribution as well as other energies developed using MATLAB, which also highlights the weak points in the drill string. These findings provide a deeper understanding of wave dynamics, leading to improved structural integrity, enhanced drilling performance, and a reduced risk of mechanical failure during drilling operations [13, [14].

Materials

This section outlines the drill string specifications, laboratory equipment, and the testing setup used to obtain experimental data relevant to the study of wave propagation under axial loading.

Drill String Specifications

The various parameters of the drill string used in this study are listed below in Table 1. They are essential for numerical simulations of wave propagation dynamics, characterization, and drill string dynamic responses to external time-varying axial forces.

These specifications were used as input for finite element simulations aimed at capturing drill string behavior under multiple load cases, specifically wave dynamics due to a time-variable axial force:

Table 1: Parameters of drill string

Parameter	**Value**(unit)	Description
Length (L)	2317 m	Total length of the drill string
Outer Diameter (OD)	0.1651 m	The outer diameter of the drill string
Inner Diameter (ID)	0.06985 m	The inner diameter of the drill string
Mass	319688 m	Total mass of the drill string
Density (ρ)	7850kg/m³	Material density is crucial for assessing mass distribution and inertia along the drill string.
Poisson's Ratio	0.3	The dimensionless ratio that describes material deformation characteristics
Stiffness	6000000 N/m	Axial stiffness, representing the drill string’s resistance to elastic deformation under loading

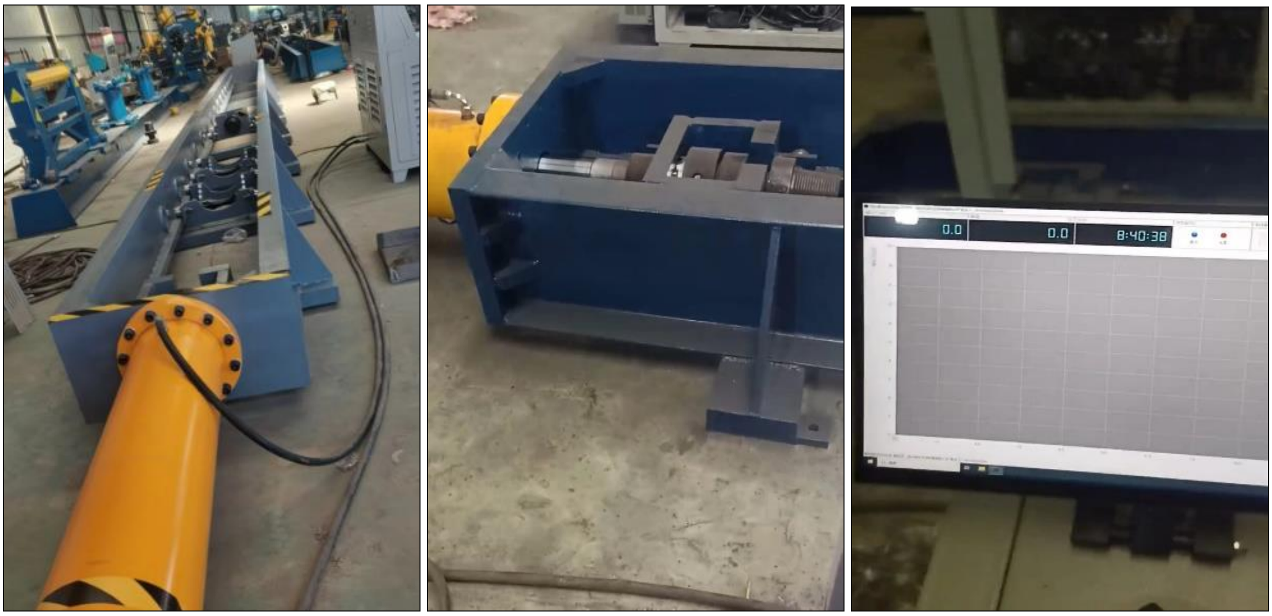
The drill string's behavior was simulated under various loading combinations by varying the axial force as a function of time within this up-and-down motion to study its effect on wave propagation dynamics. The data informs us about how the string vibrates in response and enables an accurate analysis of wave propagation phenomena through such forces.

Laboratory Equipment and Experimental Setup To study the behavior of the drill string due to time-varying axial forces, the Galip Drilling Jar Tester is used, as shown in Fig 5. This state-of-the-art equipment is designed to evaluate the load-carrying capacity of drilling jars under impact conditions that realistically simulate those encountered in field operations. Galip Drilling Jar Tester is a vital tool for understanding the effects of different axial loads on wave propagation, and it can assist in identifying similar issues during drilling.

Basic Description of the Galip Drilling Jar Tester:

- **Advanced Sensors Acquisition System:** It features a specialized clamp system on both sides of the drilling jar, allowing for precise sensor readouts of force and displacement. A drawstring encoder measures the drop distance between both clamps to provide accurate readings.
- **Pressure Transducer:** Measures force applied in psi while converting it to lb, which then accurately tracks the time-dependent kinetic energy generated by a Jar on a drilling sleeve.
- **Automatic Test Capability:** The tester´s software is adaptable to up to 6 tests, allowing for increasing and descending sequences in automatic mode with no manual intervention, ensuring uniform test outcomes.
- **Webnoizerprehensive Reporting System:** After every test sequence is complete, the system automatically saves all data collected through extensive reports, providing real-time feedback on the performance of the drill jar.
- **Durability and Reliability:** The Galip Drilling Jar Tester is engineered to withstand harsh workshop environments with a year-round OEM warranty. It uses extended-life parts, allowing it to be used under various environmental situations.

Photos (a), (b), and (c)) were taken during the test process for documentation of setup as well as to show different phases of the experiment. The Galip Drilling Jar Tester provided insightful high-resolution data on force transmitted by the drill jar in response to time-varying forces imparted for wave propagation along the string. This arrangement allowed the examination of the time-dependent force effects on wave phenomena and stress distribution to fulfill its ultimate purpose: wave action study affecting the technical status of a drill string during the drilling operation.



a. the test bench top. **b the test bench bottom.** **c the monitor screen.**

Fig 1: The laboratory test process of the drilling jar

Experimental Report and Results The testing procedure adhered to standard protocols for assessing the effects of time-varying axial forces on drill strings. A comprehensive report, documenting both the measured data and the subsequent analysis, was generated based on the outputs from the Galip Drilling Jar Tester. The primary result from the experiment was the time-varying force data captured in the report, represented in Fig 1. This force profile, derived directly from the experimental setup, will be utilized in subsequent mathematical modeling and MATLAB coding to simulate the dynamic response of the drill string under real-world loading conditions.

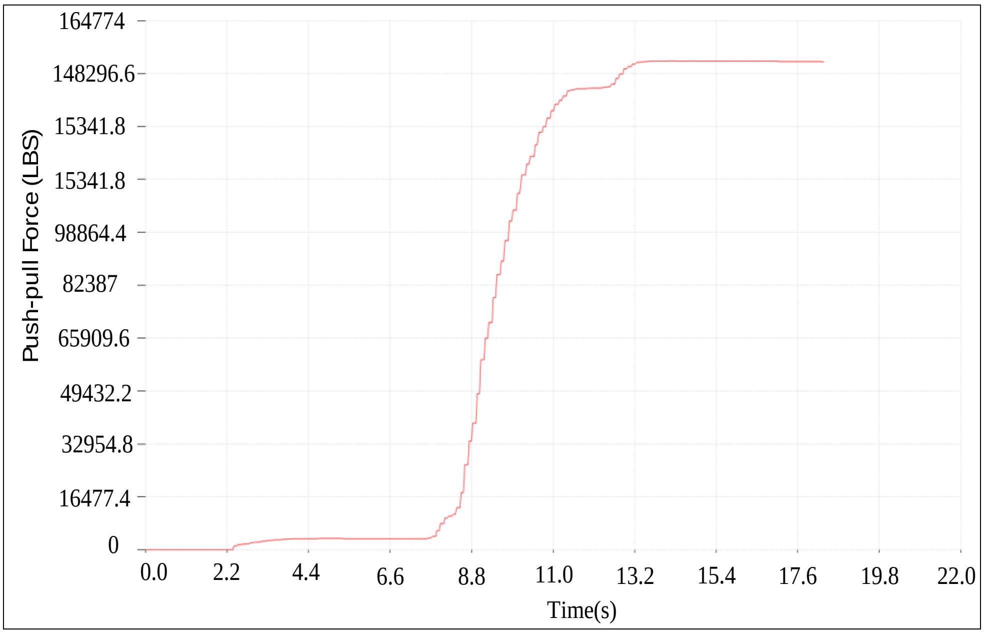


Fig 1: *Time-varying axial force.*

The results of this study give an initial insight into wave phenomena and material behavior under time-dependent axial loads on drill strings. The time-varying force profile recorded in the experimental report, as shown in Fig. One forms the lower bound of mathematical analysis and MATLAB-facilitated simulation to address variable-force demand drilling, aiming to improve operational efficiency and structural life in this loading scenario.

Mathematical Formulation with MATLAB Code Integration

The study presents an equation of motion and a displacement-based model for wave propagation in an axially loaded drill string. The thermodynamical force profile describes a substantial part of this and is used as input for the displacement analysis to simulate other dynamic loading situations realistically.

Damped Oscillation Model

The movement of the drilling string follows the motion equation for a damped dynamical system:

$$m\frac{d^2u}{dt^2}+c\frac{du}{dt}+ku=0 \tag{1}$$

Where m is the mass of the drill string segment, c is the damping coefficient, and k is the stiffness constant, the general solution to this differential equation involves a decaying exponential term combined with a cosine term:

$$u(t)=A_0e^{-\gamma t}\cos(\omega_d t+\phi) \tag{2}$$

Here, γ Is the internal friction, ω_d is the damped frequency, ω_n is the natural frequency.

Incorporating Distance Effect (x):

Taking into account the propagation of waves over position x by introducing terms that represent time delay and attenuation due to it as follows. This correction makes the equation adequately describe how waves of any type travel in a pulser through all drill strings, considering both wave travel time and position:

$$u(x,t)=A_0e^{-\gamma(t+\frac{x}{c})}\cos(\omega_d(t-\frac{x}{c})+\phi) \tag{3}$$

where $\frac{x}{c}$ represents the time it takes for the wave to reach point x.

Adjusting for Material Properties:

Wave speeds in the drill string are found to be sensitive to material properties such as Poisson's ratio ν, Young's modulus E, and density ρ. These factors determine how a material deforms under stress, affecting its wave speed and damping. The correction factor is based on these properties that alter the oscillation amplitude. This normalization ensures that the model correctly captures how the material behaves under various loads, taking into account elasticity and inertia effects on wave propagation:

$$u(x,t)=A_0e^{-\gamma(t+\frac{x}{c})}\cos(\omega_d(t-\frac{x}{c})+\phi)(1-\upsilon\frac{\rho}{E}) \tag{4}$$

Complete Displacement Equation with Time-Varying Force: The first model describes the piecewise linear representation of the primary force profile F(t) (shown in Fig 1), extracted from an experimental setup known as the Galip Drilling Jar Tester. It helps to mimic real-world scenarios at set points during time as a founding solution for accurate simulation of dynamic mechanical behavior drills string under variable time-dependent loading conditions:

$$F(t)=\begin{cases} 0 & \text{if } 0\le t<2.4 \\ 1000\times\dfrac{t-2.4}{2.5-2.4} & \text{if } 2.4\le t<2.5 \\ (4000-1000)\times\dfrac{t-2.5}{3.7-2.5}+1000\times4.44822 & \text{if } 2.5\le t<3.7 \\ 4000 & \text{if } 3.7\le t<7 \\ (145000-4000)\times\dfrac{t-7}{11.3-7}+4000\times4.44822 & \text{if } 7\le t<11.3 \\ (146880-145000)\times\dfrac{t-11.3}{12.6-11.3}+145000 & \text{if } 11.3\le t<12.6 \\ (142258.4-146880)\times\dfrac{t-12.6}{13.2-12.6}+146880 & \text{if } 12.6\le t<13.2 \\ 152258.4 & \text{if } t\ge13.2 \end{cases} \qquad (5)$$

This force function is applied in the displacement model by defining the amplitude $A_0=\dfrac{F(t)}{K}$, giving the complete displacement equation:

$$u(x,t)=\frac{F(t)}{k}.e^{-(\zeta+\gamma)(\omega_n t+\frac{x}{c})}\cdot\cos(\omega_d(t-\frac{x}{c})+\phi)\cdot(1-\upsilon\frac{\rho}{E}) \qquad (6)$$

where:ζ is the damping ratio,γ represents internal friction damping,c is the wave speed, and F(t) is derived from the experimental data to simulate real-world loading.

Cumulative Work Calculation:

The work done by the time-varying force can be calculated by integrating the product of the force $F(t)$ and the displacement $u(x,t)$ over time. The following equation represents this:

$$W(x,t)=\int F(t).u(x,t)dt \qquad (7)$$

Where $W(x, t)$ is the work taken in drilling from 0 to x at time t, F(t) denotes the axial force applied on the drill string, and $u(x,t)$ shows the displacement induced at position x and time t. This equation represents the cumulative work or energy done on the drill string as a force acts over time. Through numerical integration of this product, we can assess the amount of energy transferred over different sections along a drill string.

Simulation and Analysis in MATLAB: Using the displacement equation, in addition to the cumulative work formula, both $u(x,t)$ and $W(x,t)$ are calculated at two points on a drill string, at x = 0 and 2257 m. These calculations are driven by the time-varying force, allowing for the visualization of wave propagation and dissipation (or work accumulation) in real-time.

Displacement Analysis: Displacements over time at every point show how the hammer stacks bend under axial loading.

Captured Work: The MATLAB code calculates the work done at both points by integrating force and displacement over time, providing clues to the energy exchange on the drill string.

Based on the damped harmonic model for drill string vibration, which accounts for the effects of stroke-dependent material properties and wave propagation in coupled tension-compression axial response, this approach establishes a realistic simulation framework to investigate drill string transient behavior resulting from dynamic loading. Using the time-dependent force data in the test report to present actual working conditions, this study facilitates a comprehensive evaluation of mechanical strength and energy efficiency during drilling operations.

Results and Discussion

The MATLAB simulation, implemented using time-varying forcing to examine longitudinal wave propagation along the drill string, has yielded valuable insight into this behavior. A better understanding of how the waves behave under these varying force conditions allows us to comprehend the drill string's dynamic response.

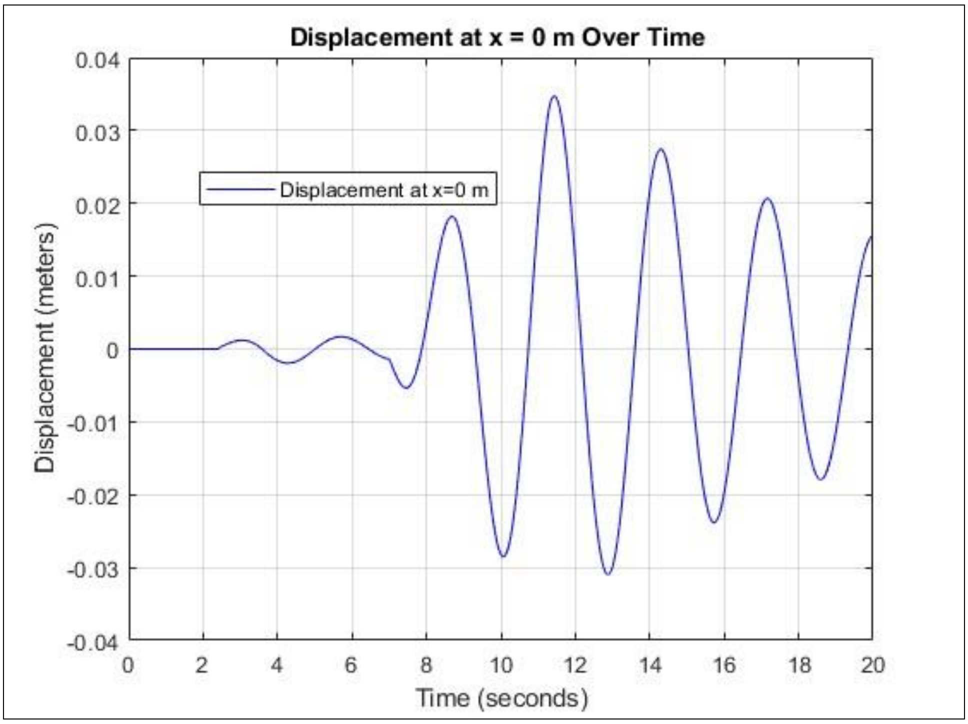


Fig 3: Displacement (m) over time at x=0 m.

Fig 4: Displacement (m) over time at x=2257 m.

Displacement Analysis At x = 0 Meters (Fig 2) A peak amplitude of around 0.03 meters is observed in the displacement at the impact site. The damped, oscillatory behavior of displacement with diminishing amplitude follows this. The high amount of energy at the impact point initially propagates in a highly concentrated form, as indicated by the prominent peak, and then starts to disperse along the drill string. The energy absorbed by the circular volume of material surrounding it and thus resisting impact is a critical indicator of how fast the drill string responds to the force pulse.

At x = 2257 Meters (Fig 3), this far-field displacement shows a lagged response, peaking around 0.025 meters in amplitude. There will be more damped versions of these oscillations in the future. The decrease in amplitude of the wave as it propagates away from its point of impact indicates a loss of energy along the drill string. Such a delayed and dampened response suggests that the underlying material properties, such as internal friction (γ) or elastic resistance, likely influence wave propagation.

A comparison of the displacement pattern between x = 0 and x = 2257 m provides additional insights into how the drill string attenuates waves. This entire length of drill string acts as an energy damper, causing the wave to lose its strength over the distance traveled. These findings provide essential insight into how impact forces are transmitted to the string and a quantification of the change in energy distribution around their origin. This is important for evaluating the physical structure and operational effectiveness, as well as determining whether the transferred energy was sufficient to potentially address a matter with stuck pipes, thereby increasing overall drilling efforts.

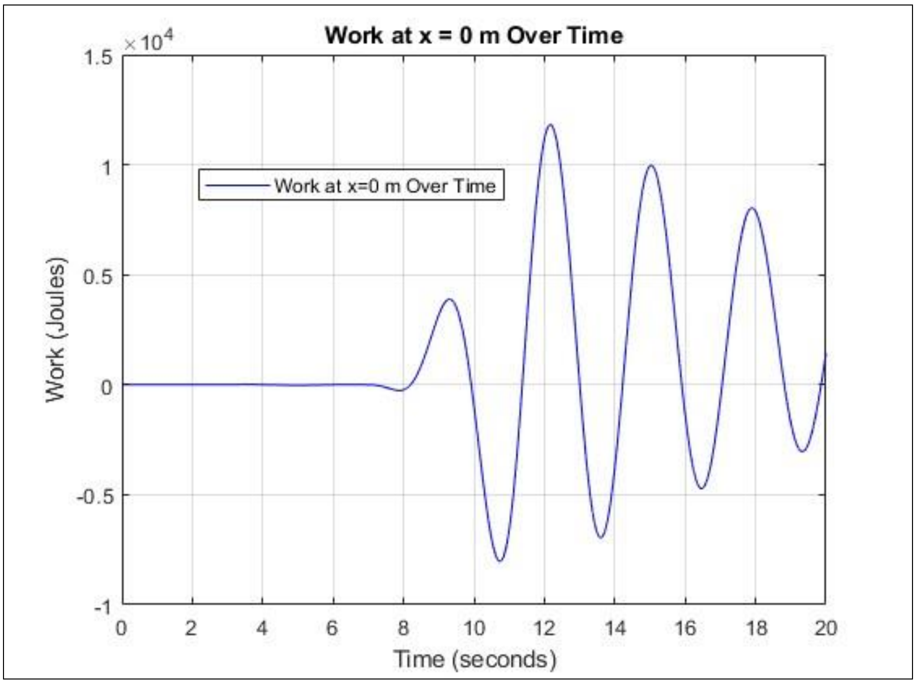


Fig 5: Work (Joules) over time at x=0 m.

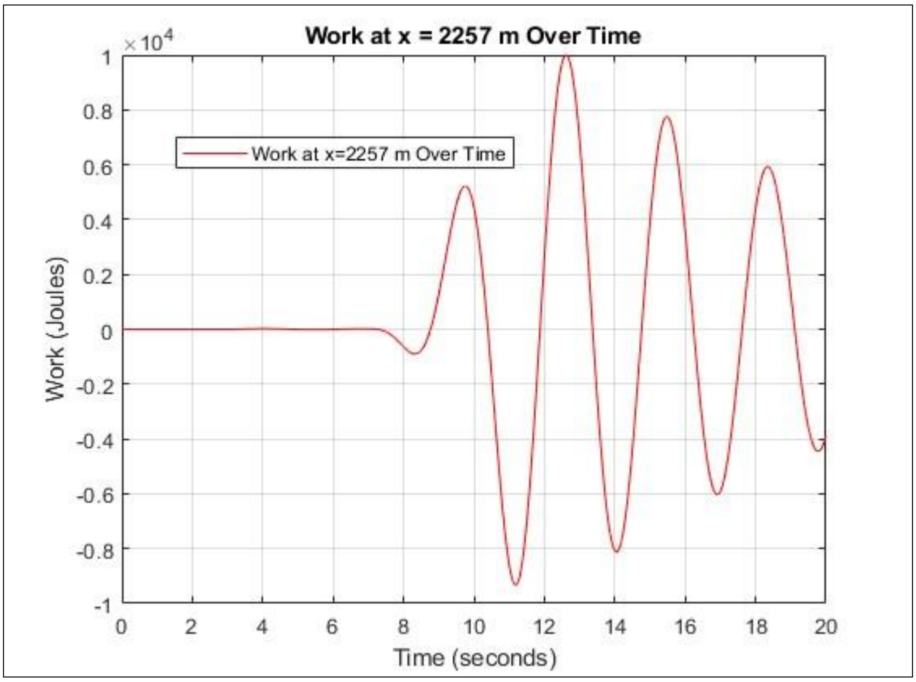


Fig 6: Work (Joules) over time at x=2257 m.

Accumulated Work Analysis (Fig 5) The accumulated work at the impact site displays substantial initial oscillations, with peak values of around 1×10^4Joules. This oscillatory behavior indicates the cyclical period for energy absorption and releases as a function of force pulse, with a significant amount of energy being captured near the point of impact. Corresponding to the sharp force-displacement response, high initial work values show that a substantial amount of energy is absorbed when this region oscillates under varying conditions.

Distance location analysis (Fig. *6*) Conversely, at x=2257 m, caution is observed in the oscillatory behavior of work with lower maximum values close to0.8×10^4 Joules. This later delay in oscillations is indicative of how long it takes a wave to come here and drop off energy; the lower work values are in agreement with the attenuation of displacement magnitude, indicating that as (work) energy is absorbed and dissipated by interactions occurring within a wave moving through the drill string due to internal friction.

Energy Propagation Along the Drill String: The total work recorded at both locations can illustrate dissipation over time in the energy distribution along a drill string through which the wave is propagating. The decreasing amplitude of work done with distance demonstrates the material's ability to absorb and dampen shock forces. This feature is indispensable for comprehending energy propagation within the drill string, as it emphasizes that material properties influence wave intensity sustainment along its progress.

Applications to Wave Performance in Drilling. This work is essential for designing and analyzing a drill string, which leads to more insight into how waves can perform throughout the whole length when an applied force from a drilling jar is used. It illustrates the effect of this force on the location of the pipe sticking. This study provides a deeper insight into energy distribution and enhances the overall understanding of how impact forces propagate through the drill string to reach far more locations down-hole, including, but not limited to, sticking points. Furthermore, these learnings are essential for maintenance and operational planning as they offer a glimpse into how waves travel down to points like sticking points of drills while interacting with the drill string structure. Displacement and work observations provide the basis for understanding the conditions of wave transmission in actual drilling scenarios, as it is necessary to design tools and techniques that efficiently pass waves within the drill string, with enhanced performance response at critical points.

Conclusion

This study demonstrates how energy from the applied force due to a drilling jar impacts through strings, concerning propagation and distribution in several points along the length of the string. Therefore, through looking at displacement and accumulated work both close to the end of impact and away from this site we can see that an oscillation is excited by force applied which damps with time due to characteristics within material itself leading wave intensity loss occurring as you move further away. The results of this study contribute to a better understanding of drill string behavior under impact loading conditions and the significance of material characteristics in energy attenuation and its management. With this, the study establishes an indispensable framework for designing practical wave energy harvesting techniques and tools within the drill string to dramatically improve drilling performance, focusing on critical events such as sticking locations.

Moreover, although this analysis provides an essential insight into mechanical response to impact forces, an in-depth parametric study is advised for studying the thermal effects on wave performance with other drill string properties; heat due to high energy impacts may result in changes in material properties that can be as Young's modulus, density, etc. The wave speed and energy dissipation features can be influenced by elevated temperatures, which may affect the overall performance of a drill string. Future work may use thermal modeling to examine the effects of temperature changes on wave propagation and energy absorption, providing complete results for some operational conditions during drilling. This would lead to more robust designs and better performance in hot drilling environments.

Reference

1. Zhang Y, Jia X, Liu Y, Gao E. Waves in elastic bars under axial constant velocity loading. International Journal of Mechanical Sciences, 2024. https://doi.org/10.1016/j.ijmecsci.2024.109230.
2. Liu W, Ni H, Wang Y, Luo Y, Gao Y, He P. Dynamic modeling and load transfer prediction of drill-string axial vibration in horizontal well drilling. Tribology International, 2022, 107986. https://doi.org/10.1016/j.triboint.2022.107986.
3. Namuq MA. Simulation and modeling of pressure pulse propagation in fluids inside drill strings,2012. Available from: https://core.ac.uk/download/pdf/235261397.pdf.
4. Ma T, Zou J, Chen P, Liu H. Investigation on the influence coupling drilling fluid and formation boundary on acoustic wave propagation in the drill string. Journal of Petroleum Exploration and Production Technology, 2020. https://link.springer.com/article/10.1007/s40948-020-00160-2.
5. Li H, Meng Y, Li G, Zhu L, Li Y, Chen Y. Effects of suspended solid particles on the propagation and attenuation of mud pressure pulses inside drill string. Journal of Natural Gas Science and Engineering,2015:22:1–10. https://doi.org/10.1016/j.jngse.2014.12.016.
6. Sarıbay M. Effect of stress wave propagation phenomenon on the determination of strain energy density theory parameters and dynamic J-integral. Fatigue & Fracture of Engineering Materials & Structures, 2022. https://doi.org/10.1111/ffe.13848.
7. Cayeux E, Jettestuen E, Ambrus A. Dynamic torque and drag model coupled with transient hydraulic. Proceedings of the ASME International Conference on Ocean, Offshore and Arctic Engineering,2 023. https://doi.org/10.1115/OMAE2023-104704.
8. Keer LM, Hou Z. Symbolic computation of longitudinal impact wave in elastic rods. Journal of Engineering Mechanics,2000:126(9):999–1005. https://doi.org/10.1061/(ASCE)0733-9399(2000)126:9(999).
9. Hu B, Eberhard P. A finite element simulation of longitudinal impact waves in elastic rods. Lecture Notes in Applied and Computational Mechanics,2011. https://link.springer.com/chapter/10.1007/978-3-642-22700-4_1.
10. Drumheller DS, Knudsen SD. The propagation of sound waves in drill strings. Journal of the Acoustical Society of America,1995:97(4):2116–2125. https://doi.org/10.1121/1.412004.
11. Drumheller DS. Acoustical properties of drill strings. Journal of the Acoustical Society of America,1989:85(3):1048–1058. https://doi.org/10.1121/1.397488.
12. Namuq MA, Reich M. Laboratory experiments on pressure wave propagation in drill strings (mud pulse telemetry). Journal of Petroleum Science, 2010. https://www.osti.gov/etdeweb/biblio/21342172.
13. Kharchenko Y, Hutyi A, Haiduk V. The influence of friction forces on longitudinal wave propagation in a stuck drill string in a borehole. Polish Maritime Research, 2018. https://t.tribologia.eu/resources/html/article/details?id=184164&language=en.
14. Rangkuti MA, Karam R. Conceptual challenges with the graphical representation of the propagation of a pulse in a string. Physical Review Physics Education Research,2022:18(2):020119. https://doi.org/10.1103/PhysRevPhysEducRes.18.020119.