



## Development of a mathematical model in python to design a drillstring with options for a given well trajectory

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### Abstract

Drillstring design is essential to the success of a drilling operation. To hit a given target, the drillstring must be able to withstand all possible loads or forces anticipated to occur during the drilling operations. Failure of any of its components will result in high wellbore restoration cost. Selection of drillstring for a given field is therefore optimised by the selection of appropriate grade of pipes, outer and inner diameter, nominal weight per foot, adequate yield and tensile strength to give the best performance in a drilling operation. The selection of these parameters manually can be very challenging and where software packages are available may be relatively expensive and limited in some applications. This work therefore seeks to develop a mathematical model that can simulate and predict recommended drillstring design boundaries for a given drillstring dimensions, well trajectory and drilling parameters to select the best drillstring design for a given drilling operation using Python programming language. Drillstring specification data from API RP 7G was used to create a database for drillstring specification. Sets of correlations from SPE papers and other literature were used to create mathematical models to design the drillstring. Three mathematical models were developed; trajectory model, drillstring specification model and algorithm model for simulation and analysis of the drillstring. The programme automatically generates two additional (upper and lower designs of the input design parameters) for comparison purposes. The rating of performances of each designed model is scored against some testing criteria like collapse, tension, torsion, torque, pipe stretch, critical buckling force and critical RPM.

**Keywords:** drillstring, design, drilling, pressure, simulation, model

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### Introduction

The drilling of an oil and gas well is a capital-intensive operation. The failure of a bottom-hole assembly (drillstring) often exponentially increases the overall cost of the drilling operation due to remedial works cost and increases in non-productive time. Approximately 75% of all lost-time incidents of more than 6 hours are caused by drilling-mechanics failure activities [1]. The majority of drillpipe failure is attributed to fatigue due to fluctuation of hook load or weight on bit. The analysis of the drillpipe buckling load has been a challenge since the buckling load can increase the bending stress and overtime leading to fatigue failure of the drill pipe [2, 3].

To design a drillstring means determining the right length, weight, grade of drill pipe to use in a drilling or coring operation. It is dependent on hole depth and size, mud weight, design safety factor, margin of pull, length and weight of drill collars, pipes and heavy wall drill pipes. A good drillstring design can improve drillstring performance with appropriate tensile and torsional strength [4]. Various types of drillstrings are deployed in a borehole for exploration and production of hydrocarbons. According to Schuberth et al [5], drillstring is made of drillpipes, drillcollars which may be instrumented to obtain measurement while drilling, stabilisers, reamers and other components needed for a specific drilling operation.

The downtime and fishing activities to recover failed drillstrings are major risks and inconveniences that drillers and operators face during drilling activities. These situations and extra cost can be minimised with proper planning of wells by the use of simulation software to analyse the performance and limitation of a designed drillstring in different drilling conditions [6].

Most drillstring commercial simulation software tools are relatively expensive to acquire and or upgrade for purposes studies and possible discovery of new findings or improvement in the understanding of the drillstring assembly [7]. Also, manual designs may be associated with errors due to tedious and long computations. Therefore, to keep the drilling crew well informed ahead of time of all potential pipe failure issues, this research develops a mathematical model that can simulate and predict basic recommended drillstring design boundaries for a given drillstring dimensions, well trajectory and drilling parameters to select an optimal drillstring design for a given drilling operation using Python programming language. This will help drilling crews to easily design and anticipate drillstring design limits. The model will serve as an easy and relatively inexpensive model that can easily be updated or modified compared to existing commercial software packages. This work contributes to the fulfillment of the 9th Sustainable Global Development goal by the United Nations; Improving Industrial Innovation and Infrastructure.

## Methods and Materials

Python programming language was used to develop a mathematical model with three sub-models (trajectory, drillstring, and control algorithm). It then simulates drillstring design limitations for three designs based on a single input design. Comparison is done for the three designs based on some seven basic design factors such as collapse, tension, torsion, torque, stretch, critical buckling force, and critical rpm. optimum drillstring design specification (limit) from the analysis of the simulation results is therefore proposed.

## Well Trajectory Model

The trajectory model receives the target coordinates of a well (northern, eastern and depth, TVD). Other input parameters are Kick-Off Point (KOP), rotary table height above mean sea level, inclination of well at end of build, build-up rate and bearing. The following trajectory equations and diagram adopted from [3, 6, 8]. The output of this model includes the planned well trajectory, measured depth, the position of the rig and its departure from the target. The measured depth would be used in the determining the actual length of the drillstring. This necessitates the developing of the trajectory plan before the drillstring design. Figure 1 shows a well trajectory of a directional well.

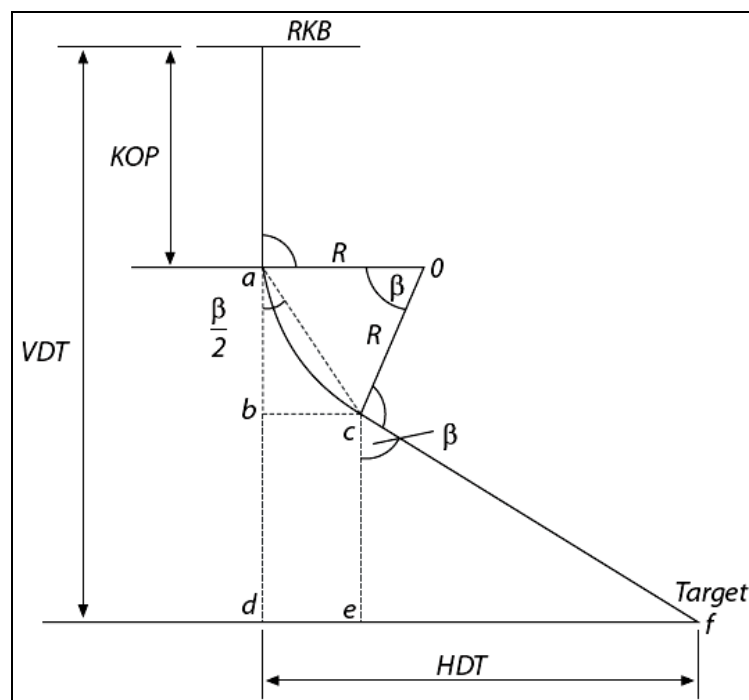


Fig 1: Direction Well Trajectory [6]

## Trajectory Simulation

A test case well trajectory was developed from the following input data:

Rotary table height = 100 ft

Kick-off point = 2 500 ft

Build-up rate = 1.2 °/100 ft

Inclination at end of build = 20 °

Well target coordinates = (5 249 ft, 4 593 ft, 10 000 ft)

The trajectory model determined the coordinates of the vertical section of the well to the kick-off point using a pre-selected depth interval; for this case 10 ft. Using the build-up rate, it determined the radius of build and the coordinates of the build section till the inclination at the end of build is equivalent to the specified inclination. The well path coordinates for the hold section were calculated with constant inclination till the target was reached. Other parameters calculated were measured depth, departure of rig from target and the rigs position.

## Drillstring Design Model

This model contains a database of drillstring parameters recommended by America Petroleum Institute (API RP7G). The two types of API pipe grades are considered in this work, Grade D Grade E drill pipes (Table A1). Figure A1 shows the flow chart of the process. Different grades of drill pipes have different yield strength and this affects the choice of drillstring grade in drillstring design. In developing the model, the inner and outer diameter of a drill pipe is selected and other related data is automatically added from the database for the simulation. The mud weight, horsepower to rotate the drillstring, rotation speed, torque and shear stress are specified or modelled for simulation. Table 1 shows the three drillstring design parameters used in the simulation. Grade D and E have minimum yield strength of 55,000 and 75,000 psi respectively [3, 6].

**Table 1:** The Three Drillstring Design Parameters Used in The Simulation

Drillstring Design	Outer Diameter (inches)	Inner Diameter (inches)	Nominal Weight (lb/ft)
Selected Drillpipe	4	3.340	14.00
Alternative 1	4	3.476	11.85
Alternative 2	4.5	3.826	16.60
Drill Collar	7	3.25	102.5
Bit Size	7.5		

### Algorithm Model (Simulation)

The algorithm model retrieves trajectory data and drillstring specification data from the aforementioned models. It analyses the operational behaviour and compares it with the design limitations defined in the database. The algorithm model also selects two other alternative drillstring designs by choosing other two closest outer and inner diameter variations from the database for analysis and comparison with the selected drillstring design. This can point the drilling crew in the right direction in selecting a new drill pipe (outer and inner diameter specifications) if the current drillstring under analysis fails the test. The algorithm generates the coordinates for the well plan and plots it. It simulates the drillstring performance and limitations and plots related graphs for analyses. By comparison, the too-deviated design(s) among the selected design and the other two alternatives can be identified. The drillstring that passes the analysis is further studied for other operational parameters such as torque, stretch of drillstring due to its own weight. The neutral point of drillstring and critical buckling force are also considered. According to [3, 6], the neutral point is a part of the drillstring which is periodically under tension and compression while the critical buckling force is the compressive load at which the drillstring will buckle. The simulation processes included tensile yield, collapse pressure, critical rpm and torque analysis. All the equations used in the model development are shown in Table A2.

Collapse pressure is the pressure above which the drill pipe will collapse. This happens when the annulus pressure exceeds the pipe's maximum collapse pressure. A design safety factor of 1.125 was used. Tensile strength of the pipe specifies the maximum load the drill pipe can support without failing. From [6], the weight of drillstring should not exceed the tensile strength of the drill pipe at the top of the drillstring. The torque generated from rotating the drillstring should not exceed the minimum required torque of the tool joint. This prevents the failure of the drill pipe at the tool joint. Critical RPM is the rotation per minutes at which drill pipe whirling will occur. Whirling of drill pipe increases its vibration and chances of pipe failure due to excessive vibration.

Tensile Yield Analysis: To account for the buoyancy effect by the drilling fluid and safety factor, the expected drillstring weight is multiplied by the buoyancy factor of the drilling fluid and the design safety factor specified. To account for extra force applied to free stuck pipe, a Margin of Overpull (MOP) was added to the expected load. This will allow the pulling of the stuck pipe without causing failure of the drill pipe. In this design, a margin of overpull of 100,000 lbf was used. The buoyancy factor of 0.847 from 10 ppg of mud and design safety factor of 1.125 was used. The safety factor reduces the risk of tensile failure of the drillstring due to unpredictable conditions or events. Due to the biaxial loading, the drill pipe is stretched resulting in a decrease in its collapse resistance [3,6].

Torsional and Yield Analysis were calculated and compared with the allowable torsion of the drill pipe from the database for analysis. The simulation provided drilling operation information of the horsepower required to rotate the drillstring with the associated torque. It compared the simulated torque with the make-up torque of pipe tool joint. This ensures that the torque produced by the rotation of the drillstring does not exceed the minimum torque that will cause the drillstring to fail at its tool joint as the tool joint is known to be weakest part of the seamless pipe [3, 6]. The Critical RPM for Whirling; the critical rotational speed at which the drill sting will vibrate (whirling) was calculated in order to keep the rotational speed below this limit.

Compressive and Tension Loading of Pipes: During drilling, it is advisable to run the drill pipes in tension. Running drill pipes in compression while rotating leads to high chances of drill pipe buckling and pipe failure. To run the drillstring in tension, the buoyed weight of the drill collars must exceed the buoyed weight of the drilled pipes. The simulation performed analysis on the critical buckling force (compressive loading that will cause buckling), the allowable weight on bit above which buckling will occur. This was compared with the planned maximum weight on bit to be used during the drilling operation.

Neutral Point: Typically, the drill collar lengths are selected such that the neutral point is 85% of the total length of drill collars. The rationale is that with a 15% safety margin, the bouncing of the bit on the bottom of the hole will prevent the neutral point from moving up into the drill pipe. If the neutral point moves up into the drill pipe, the rotating thin wall drill pipe buckles and can fail prematurely [3, 6, 9]. The simulator designs the drillstring such that the neutral point falls in the length of the drill collar. The heavy wall and stiffness of the drill collar enables it to withstand the periodic compressional and tensional loading. Downhole devices such as downhole sensor should not be place at the neutral point to prevent the failure of such devices [3, 6].

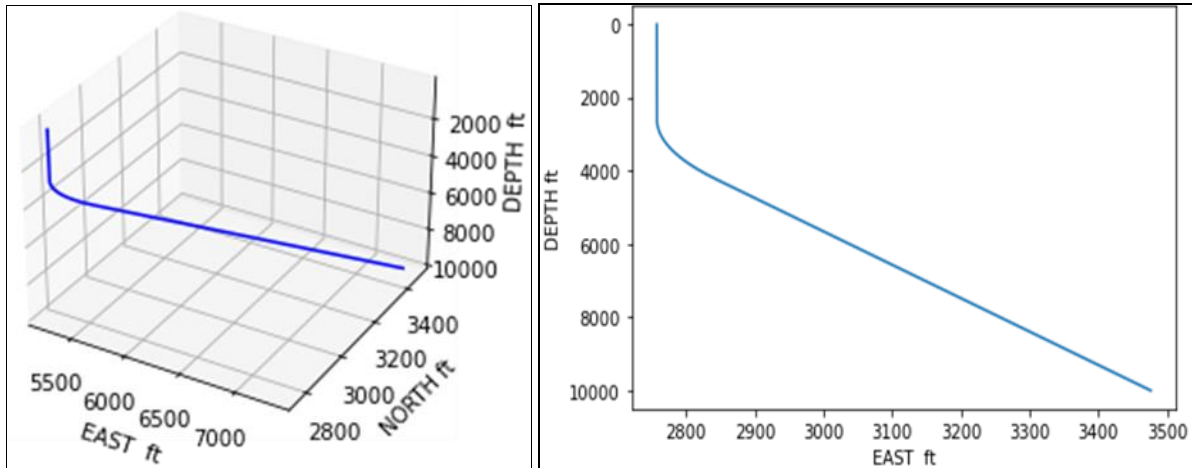
## Results and Discussions

### Trajectory Results

The trajectory model determined the rig position, measured depth, radius of build when given the target coordinates of the well, build-up rate and kick-off point. The output data from the trajectory model is shown in Table 2 and Figure 2.

**Table 2:** Output Parameters of Trajectory Model

Parameters	Values (ft)	Parameters	Values (ft)
Rig Position East	2,206	Radius	4775
Rig Position North	5,249	MD Build	1667
Rig Position Vertical	100 ft (Above MSL)	TVD Build	1633
Departure	2387	Departure Build	288
Measured Depth (MD)	10,410	MD Hold	6137
TVD	10,000		

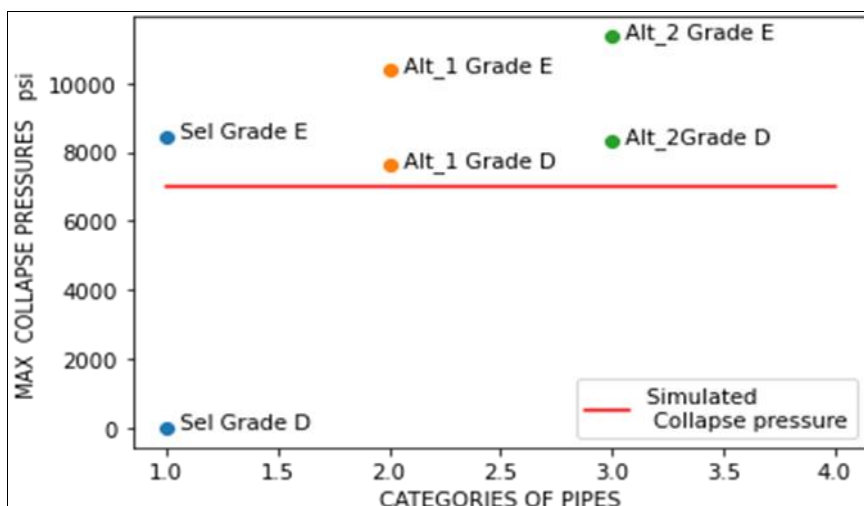


**Fig 2:** 3D and 2D Well Trajectories

The measured depth from the trajectory model was used as the total length of the drillstring. The inclination was also retrieved from the output of the trajectory model.

**Drillstring Simulation Results**

The results for the simulations of three different pipe designs are analysed and discussed in this section. The simulation results are analysed based on some drillstring testing criteria. The tendency of the drillstring to collapse is due to hydrostatic pressure by drilling mud and true vertical depth. When pressure in the annulus is greater than the collapse pressure of the drill pipe, the pipe will collapse. From Figure 3, the selected drillstring design passed the collapse pressure test for only E pipes. However, the alternatives for both pipe grades D and E passed the test. The Grade D pipe will fail under the simulated collapse pressure.



**Fig 3:** Collapse Pressure Results

Tensile load (ability of a pipes to withstand a pulling force) is shown in Figure 4 for pipe grades D and E. The results of the tensile yield strength of the two alternative drillstring designs are shown. From Figure 4, both Grades D and E will fail anticipated Total Load, Maximum Allowed Load and Load with Safety. Therefore, a depth of about 10,410 ft and high MOP value are not suitable for the current design. However, for Alternative 1 and 2 designs, Grade E will only be able to accommodate Total Load force and Total Load and Maximum Allowable Loads respectively.

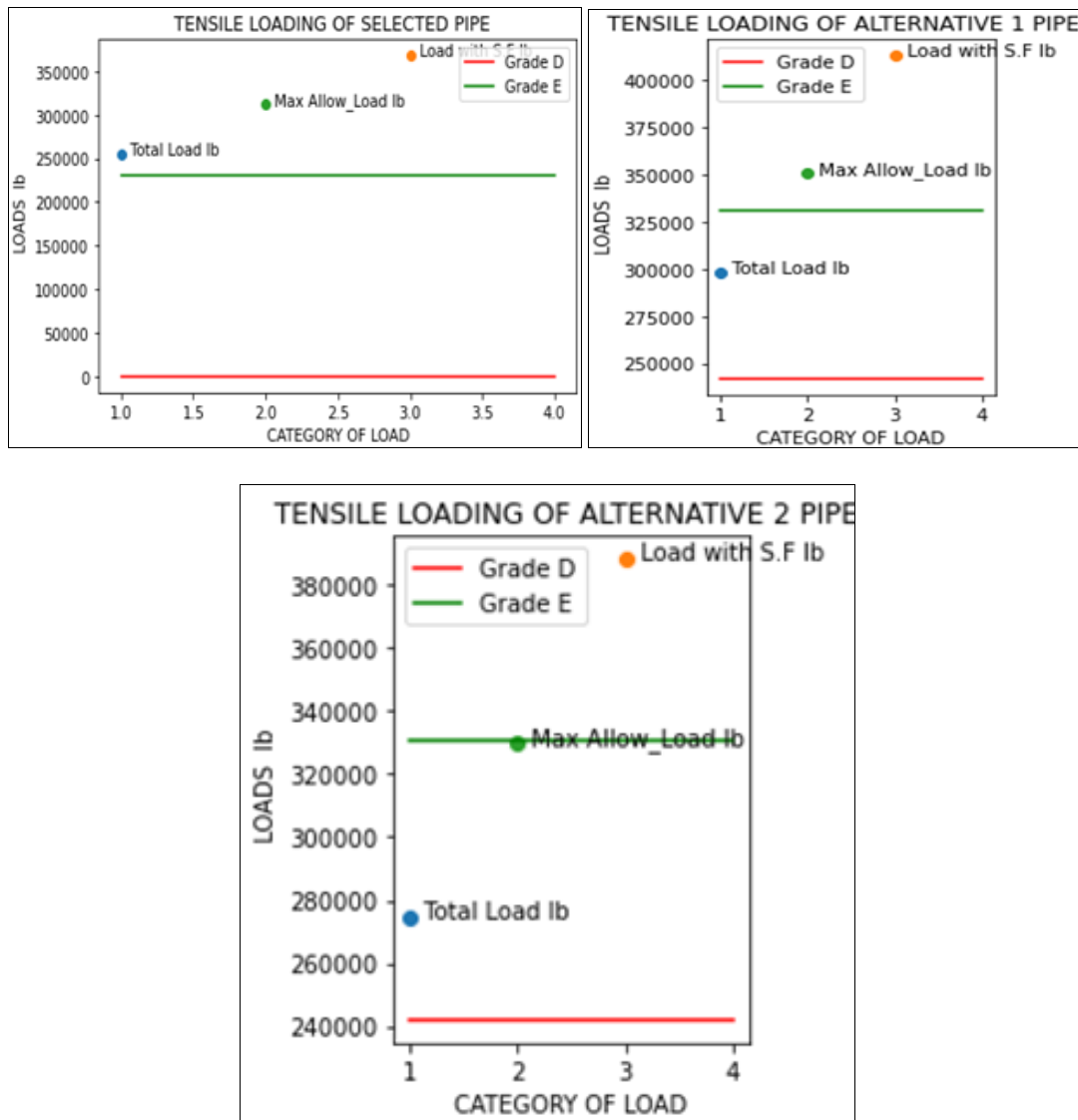


Fig 4: Plot of Tensile Loads of the Selected Drillstring

To keep the selected drillstring in tension by 85% weight of both drill collars and possible with about 20% of HWDP, a total of 124,240 lb weight will be made available by selected design (Figure 5). A better weight of 127,475 will be provided by Alternative 2 while Alternative 1 will provide the best weight on bit of 131,387 lb.

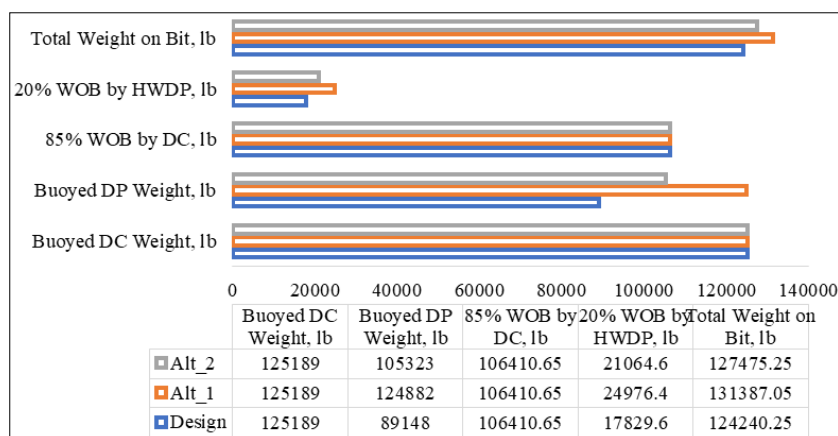


Fig 5: WOB Values by Various Designs

**Other Testing Criteria**

Figure 6 and 7 show some default and calculated values of the pipe grades limits for the Design, Alternative 1 and 2. From Figure 6, the calculated torques were all below the default torques of the various pipe grades and therefore makes them safer for use with the Design and Alternative 2 recoding the least value of 5251 psi. However, the torsion calculated was highest for Alternative 2 (11,087 ft-lbf) followed by Alternative 2 (7,132 ft-

lbf) and Design with the least values of 4,663 ft-lbf. By default, Grade D pipes do have a recorded torsion value of 0 ft-lbf therefore could not pass the test. In Figure 7, Alternative 1 produced the highest Critical Vibration, Critical Buckling values followed by Alternative 2 and lastly the Design. The monitoring of these values aids in optimisation of a drilling process. The best values were recorded by the Design option. Other parameters for analysis are shown in Table 3.

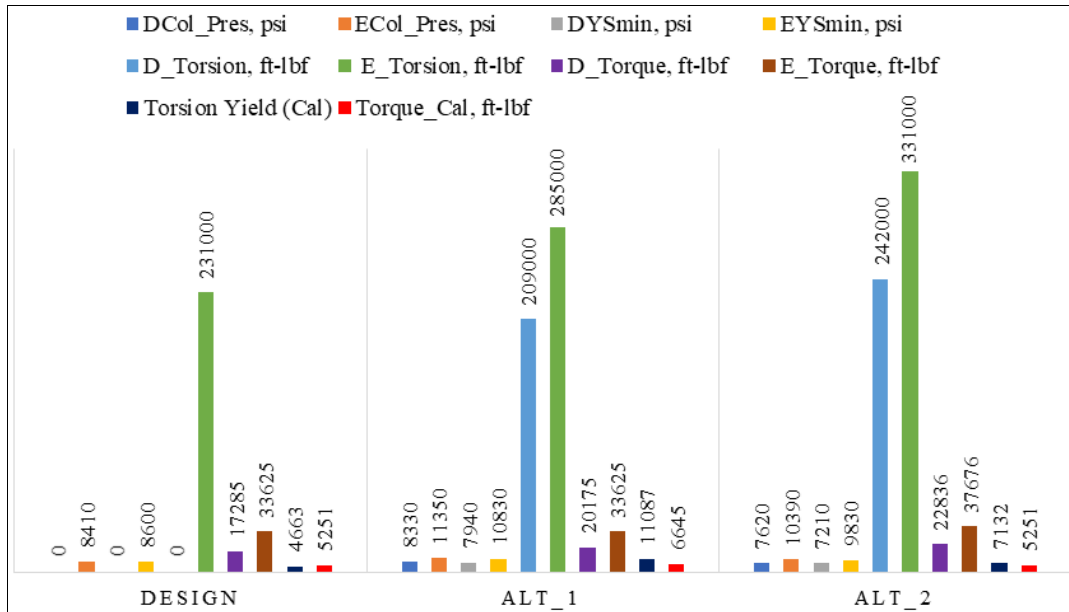


Fig 6: Default Parameters for Various Grades of Pipes

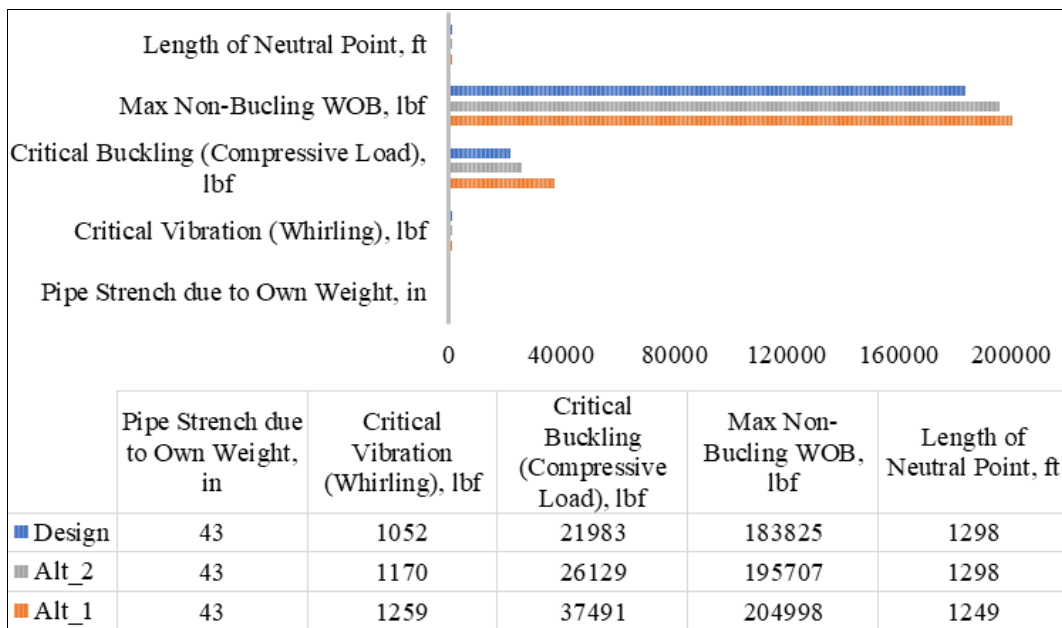


Fig 7

Table 3: Output Parameters of Model

Parameters	Design	Alt_1	Alt_2	Units
Total Load	255164	297705	274419	lbs
Load at Surface	367622	412590	388140	lbs
Max Allowable Load	312734	350701	329919	lbf
Horse Power (Rotation)	100	127	100	hp
Torque	5251	6645	5251	ft-lbf
Torsion Yield (Tension)	4663	11087	7132	ft-lbf
Pipe Stretch due to Own Weight	43	43	43	in
Max Shear Stress	2042	1782	1977	psi
Critical Vibration (Whirling)	1052	1259	1170	rpm
Critical Buckling (Compressive Load)	21983	37491	26129	lbf

Max Non-Buckling WOB	183825	204998	195707	lbf
Planned Max WOB use	111776	111776	111776	lbf
Length of Drill collar	1454	1454	1454	ft
Length of Neutral Point	1298	1249	1298	ft
Buoyed DC Weight	125189	125189	125189	lb
Buoyed DP Weight	89148	124882	105323	lb
85% WOB by DC	106410.7	106410.7	106410.7	lb
20% WOB by HWDP	17829.6	24976.4	21064.6	lb
Total Weight on Bit	124240.3	131387.1	127475.3	lb

### Conclusions

From the results gathered, it can be concluded that the mathematical model of the drillstring identified results produced by a well trajectory to design two various designs in addition of a selected design to aid in making a safe and technical decision. These testing criteria included collapse, tension, torsion, torque, pipe stretch, critical buckling force, critical rpm and vibration (traverse). This model is relative fast, cheaper and can easily be upgraded to include other design parameters.

### References

1. Burgess TM, Martin CA. "Wellsite Action on Drilling Mechanics Information Improves Economics", 1995. [https://petrowiki.spe.org/Drilling\\_dynamics#Application\\_of\\_downhole\\_shock\\_sensor](https://petrowiki.spe.org/Drilling_dynamics#Application_of_downhole_shock_sensor). Accessed: April 25, 2021.
2. Albdiry MT, Almensory MF. "Failure Analysis of Drillstring in Petroleum Industry: A Review", 2016. <https://www.sciencedirect.com/science/article/abs/pii/S1350630716300838>. Accessed: April 25, 2021.
3. Amarin R. Advanced Drilling Engineering, Unpublished Postgraduate Lecture Material, University of Mines and Technology, 2021, 171.
4. Anon. "Procedures for Drill String Design Engineering Essay", 2015. <https://www.ukessays.com/essays/engineering/procedures-for-drill-string-designengineering-essay.php>. Accessed: July 10, 2021.
5. Schubert F, Reckmann H, Macpherson JD, Hood JA. "Realtime Dogleg Severity Prediction", 2015. <https://www.freepatentsonline.com/9043152.html>. Accessed: July 10, 2021.
6. Hossain ME, Abdulaziz AA. "Fundamentals of Sustainable Drilling Engineering", Scrivener Publishing LLC, 2015, 751.
7. Losoya EZ, Gildin E, Noynaert SF, Medina-Zetina Z, Crain T, Stewart S *et al.* "An Open-Source Enabled Drilling Simulation Consortium for Academic and Commercial Applications", Latin America and Caribbean Petroleum Engineering Conference, Virtual, 2020, 86-91.
8. Amarin R, Broni-Bediako E. "Application of Minimum Curvature Method to Well-path Calculations", Research Journal of Applied Sciences, Engineering and Technology, 2010;2(7):679-686.
9. Blick EF, Shams Q. "Effect of Flow Properties on Drillpipe Neutral Point." Paper presented at the SPE/IADC Drilling Conference, New Orleans, Louisiana, 1989. doi: <https://doi.org/10.2118/18694-MS>.
10. Anon. Drilling Engineering Workbook, Unpublished Baker Hughes INTEQ, Training and Development Manual: A Distributed Learning Course, 80270H Rev., 2520 W.W. Thorne, Houston, TX 77073, USA, 1995:713-625-4415:410.
11. Anon. "Recommended Practice for Drill Stem Design and Operating Limits", America Petroleum Institute, 1998, 146.