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Image compression using Wavelets

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Abstract
Wavelets provide a powerful and remarkably flexible set of tools for handling fundamental problems in science and engineering, such as audio de-noising, signal compression, object detection and fingerprint compression, image de-noising, image enhancement, image recognition, diagnostic heart trouble and speech recognition, to name a few. Here, we are going to concentrate on wavelet application in the field of Image Compression so as to observe how wavelet is implemented to be applied to an image in the process of compression, and also how mathematical aspects of wavelet affect the compression process and the results of it. Wavelet image compression is performed with various known wavelets with different mathematical properties. We study the insights of how wavelets in mathematics are implemented in a way to fit the engineering model of image compression.

Keywords: wavelet, image, color

1. Introduction

Waveets are functions which allow data analysis of signals or images, according to scales or resolutions. The processing of signals by wavelet algorithms in fact works much the same way the human eye does; or the way a digital camera processes visual scales of resolutions, and intermediate details. But the same principle also captures cell phone signals, and even digitized color images used in medicine. Wavelets are of real use in these areas, for example in approximating data with sharp discontinuities such as choppy signals, or pictures with lots of edges. While wavelets is perhaps a chapter in function theory, we show that the algorithms that result are key to the processing of numbers, or more precisely of digitized information, signals, time series, stillimages, movies, color images, etc. Thus, applications of the wavelet idea include big parts of signal and image processing, data compression, fingerprint encoding, and many other fields of science and engineering. This thesis focuses on the processing of color images with the use of custom designed wavelet algorithms, and mathematical threshold filters. Although there have been a number of recent papers on the operator theory of wavelets, there is a need for a tutorial which explains some applied tends from scratch to operator theorists. Wavelets as a subject is highly interdisciplinary and it draws in crucial ways on ideas from the outside world. We aim to outline various connections between Hilbert space geometry and image processing. Thus, we hope to help students and researchers from one area understand what is going on in the other. One difficulty with communicating across areas is a vast difference in lingo, jargon, and mathematical terminology. With handson experiments, our paper is meant to help create a better understanding of links

Between the two sides, math and images. It is a delicate balance deciding what to include. In choosing, we had in mind students in operator

Theory, stressing explanations that are not easy to find in the journal literature. Our paper results extend what was previously known, and we hope yields new insight into scaling and of representation of color images; especially, we have aimed for better algorithms. His would allow the image to lose a certain amount of horizontal and diagonal details, but would

Not affect the image in human perception. As mathematically illustrated in 3.3, an image can be decomposed into approximate, horizontal, vertical and diagonal details. N levels of decomposition is done. After that, quantization is done on the decomposed image where different

Quantization maybe done on different components thus maximizing the amount of needed details and ignoring 'not-so-wanted' details. This is done by thresholding where some coefficient values for pixels in images are 'thrown out' or set to zero or some 'smo zero or some 'smoothing' effect is done on the image matrix. This process is used in JPEG2000.

Motivation. In many papers and books, the topics in wavelets and image processing are discussed in mostly in one extreme, namely in terms of engineering aspects of it or wavelets are discussed in terms operators without being specifically mentioned how it is being used in its application in engineering. In this paper, the author adds onto [Sko01], [Use01] and [Vet01] more insights about mathematical properties such as properties from Operator Theory, Functional Analysis, etc. of wavelets playing a major role in results in wavelet image compression. Our paper aims in establishing if not already

Established or improve the connection between the mathematical aspects of wavelets and its application in image processing. Also, our paper discuss on how the images are implemented with computer program, and how wavelet decomposition is done on the digital images in terms of computer program, and in terms of mathematics, in the hope that the communication between mathematics and engineering will improve, thus will bring greater benefits to mathematicians and engineers.

2. Wavelet Color Image Compression

Methods. The whole process of wavelet image compression is performed as follows: An input image is taken by the computer, forward wavelet transform is performed on the digital image, thresholding is done on the digital image, and entropy coding is done on the image where necessary, thus the compression of image is done on the computer. Then with the compressed image, reconstruction of wavelet transformed image is done, then inverse wavelet transform is performed on the image, thus image is reconstructed. In some cases, zerotree algorithm [Sha93] is used and it is known to have better compression with zerotree algorithm but it was not implemented here.

Forward Wavelet Transform.

Various wavelet transforms are used in this step. Namely, Daubechies wavelets, Coiflets, biorthogonal wavelets, and Symlets. These various transforms are implemented to observe how various mathematical properties such as symmetry,

Number of vanishing moments and orthogonality differ the result of compressed image. Advantages short support is that it preserves locality. The Daubechies wavelets used are orthogonal, so do Coiflets. Symlets have the property of being close to symmetric. The biorthogonal wavelets are not orthogonal but not having to be orthogonal gives more options to a variety of filters such as symmetric filters thus allowing to possess the symmetric property. MATLAB has a subroutine called wavedec2 which performs the decomposition of the image for you up to the given desired level (N) with the given desired wavelet (wname). Since there are three components to deal with, the wavelet trans form was applied component wise. “wavedec” is a two dimensional wavelet analysis function. [C,S] = wavedec2(X,N,‘wname’) returns the wavelet decomposition of the matrix X at level N, using the wavelet named in string ‘wname’. Out puts are the decomposition vector C and the corresponding book keeping matrix S [MatlabUG]. Here the image is taken as the matrix X. 2.1.2. Thresholding. Since the whole purpose of this project was to compare the performance of each image compression using different wavelets, fixed thresholds were used. Soft threshold was used in this project in the hope that the drastic differences in gradient in the image would be noted less apparently. The soft and hard thresholding Soft, hard are defined as follows:

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$$(2.1) \qquad T_{soft}(x) = \begin{cases} 0 & \text{if } |x| \le \lambda \\ x - \lambda & \text{if } x > \lambda \\ x + \lambda & \text{if } x < -\lambda \end{cases}$$

$$(2.2) \qquad T_{hard}(x) = \begin{cases} 0 & \text{if } |x| \le \lambda \\ x & \text{if } |x| > \lambda \end{cases}$$

where λ ∈ R+. As it could be observed by looking at the definitions, the difference between them is related to how the coefficients larger than a threshold value λ in absolute values are handled. In hard thresholding, these coefficient values are left alone. Unlike in hard thresholding,

The coefficient values area decreased by λ if positive and increased by λ if negative [Waln02]. MATLAB has this subroutine called wthrmngr which computes the global threshold or level dependent thresholds depending on the option and method. The options available are global threshold and level dependent threshold, and the global threshold is used in the program. However, a fixed threshold values were used so as to have the same given condition for every wavelet transform to compare the performances of different conditions. Here, fixed thresholds 10 and 20 were used. For the lossless compression 0 is used as the threshold for an obvious reason.

Entropy Encoding. Entropy defined as

$$H(s) = -\sum_{i=1}^{q} P(s_i) log_2(P(s_i)),$$

Where is are code words and S is the message. Entropy coding uses code words with varying lengths, here code words with short length are used for values that have to be encoded more often, and the longer code words are assigned for less encoded values. H(S) measures the amount of information in the message, i.e. the minimal number of bits needed to encode one word of the message. Unfortunately, the entropy encoding was not implemented on the codes for the color image compression using wavelets. However, Shannon entropy which is defined below was used in the code for the image compression with wavelet packets. See [Son04] and [Bra02]. The Shannon entropy functional is defined by

$$(2.3) \qquad M(c\{b_j\}) = -\sum_{n=1} M|\langle c, b_j\rangle|^2 log|\langle c, b_j\rangle|^2$$

Also, entropy could be viewed as a quantity that measures the amount of uncertainty in a probability distribution, or equivalently of the amount of information obtained from one sample from the probability space. Reconstruction of Wavelet Transformed Image. At this step, the significance map is taken and with the amplitudes of the nonzero valued wavelet coefficients, the wavelet transformed image is reconstructed.

Inverse Wavelet Transformation. The wavelet parameters are converted back into an image almost identical to the original image. How much identical they are will be dependent upon whether the compression was lossy or lossless Wavelets. Compactly supported wavelets are functions defined over a finite interval and having an average value of zero. The basic idea of the wavelet transform is to represent any arbitrary function f(x) as a superposition of a set of such wavelets or basis functions. These basis functions are obtained from a single prototype wavelet called the mother wavelet ψ(x), by dilations or scaling and translations. Wavelet bases are very good at efficiently representing functions that are smooth except for a small set of discontinuities. For each n, k ∈ Z, define ψn,k(x) by

$$\psi_{n,k}(x) = 2^{n/2}\psi(2^n x - k) \tag{2.4}$$

Constructing the function $\psi(x)$, L^2 on $\mathbb{R}$, such that $\{\psi_{n,k}(x)\}_{n,k\in\mathbb{Z}}$ is an orthonormal basis on $\mathbb{R}$. As mentioned before $\psi(x)$ is a wavelet and the collection $\{\psi_{n,k}(x)\}_{n,k\in\mathbb{Z}}$ is a wavelet orthonormal basis on $\mathbb{R}$; this framework for constructing wavelets involves the concept of a multiresolution analysis or MRA.

2.2.1. *Multiresolution Analysis.* Multiresolution analysis is a device for computation of basis coefficients in $L^2(\mathbb{R}) : f = \sum\sum c_{n,k}\psi_{n,k}$. It is defined as follows, see [Kei04]: Define

$$V_n = \{f(x) | f(x) = 2^{n/2} g(2^n x), g(x) \in V_0\},$$

where

$$f(x) = \sum_{n\in\mathbb{Z}} \langle f, \phi(\cdot - n)\rangle \phi(x - n).$$

Then a multiresolution analysis on $\mathbb{R}$ is a sequence of subspaces $\{V_n\}_{n\in\mathbb{Z}}$ of functions L^2 on $\mathbb{R}$, satisfying the following properties:

(a) For all $n, k \in \mathbb{Z}, V_n \subseteq V_{n+1}$.

(b) If $f(x)$ is C_c^0 on $\mathbb{R}$, then $f(x) \in \overline{span}\{V_n\}_{n\in\mathbb{Z}}$. That is, given $\epsilon > 0$, there is an $n \in \mathbb{Z}$ and a function $g(x) \in V_n$ such that $\|f - g\|_2 < \epsilon$.

(c) $\bigcap_{n\in\mathbb{Z}} V_n = \{0\}$.

(d) A function $f(x) \in V_0$ if and only if $2^{n/2} f(2^n x) \in V_n$.

(e) There exists a function $\phi(x)$, L^2 on $\mathbb{R}$, called the scaling function such that the collection $\phi(x - n)$ is an orthonormal system of translates and $V_0 = \overline{span}\{\phi(x - n)\}$.

Definition 2.1. Let $\{V_J\}$ be an MRA with scaling function $\phi(x)$ which satisfies (2.14) and scaling filter $h(k)$, where $h(k) = \langle 2^{-1/2}\phi(\frac{x}{2}), \phi(x-k)\rangle$. Then the wavelet filter $g(k)$ is defined by

$$g(k) = (-1)^k \overline{h(1-k)}$$

and the wavelet by

$$\psi(x) = \sum_{k\in\mathbb{Z}} g(k)\sqrt{2}\phi(2x - k).$$

See [Kei04].

Then $\{\psi_{n,k}(x)\}_{n,k\in\mathbb{Z}}$ is a wavelet orthonormal basis on $\mathbb{R}$.

Definition 2.2. The orthogonal projection of an arbitrary function $f \in L^2$ onto V_n is given by

$$P_n f = \sum_{k\in\mathbb{Z}} \langle f, \phi_{n,k}\rangle \phi_{n,k} \qquad L^2$$

[Kei04].

[Kei04].

3. Digital Image Representation.

An image is defined as a twodimensional function ie. a matrix, f(x, y), where x and y are spatial coordinates, and the amplitude of f at any pair of coordinates (x, y) is called the intensity or gray level of the image at the point. Color images are formed by combining the individual twodimensional images. For example, in the RGB color system, a color images consists of three namely, red, green and blue individual component images. Thus many of the techniques developed for monochrome images can be extended to color images by processing the three

Component images individually. When x, y and the amplitude values of f are all finite, discrete quantities, the image is called a digital image. The field of digital image processing refers to processing digital images by means of a digital computer. A digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels and pixels. Since pixel is the most widely used term, the elements will be denoted as pixels from now on. An image maybe continuous with respect to the x- and y-coordinates, and also in amplitude. Digitizing the coordinates as well as the amplitude will take into effect the conversion of such an image to digital form. Here, the digitization of the coordinate values are called sampling; digitizing the amplitude values is called quantization. A digital image is composed of a finite number of elements, each of which has a particular location and value. The field of digital image processing refers to processing digital images by means of a digital computer. See [Gon04].

Coordinate Convention

Assume that an image f(x, y) is sampled so that the resulting image has M rows and N columns. Then the image is of size M ×N. The values of the coordinates (x, y) are discrete quantities. Integer values are used for these discrete coordinates. In many image processing books, the image origin is

Set to be at (x, y) = (0, 0). The next coordinate values along the first row of the

Image are (x, y) = (0, 1). Note that the notation (0, 1) is used to signify the second sample along the first row. These are not necessarily the actual values of physical coordinates when the image was sampled. Note that x ranges from 0 to M−1, and y from 0 to N − 1, where x and y are integers. However, in the Wavelet Toolbox the notation (r, c) is used where r indicates rows and c indicates the columns. It could be noted that the order of coordinates is the same as the order discussed previously. Now, the major difference is that the origin of the coordinate system is at (r, c) = (1, 1); hence r ranges from 1 to M, and c from 1 to N for r and c integers. The coordinates are referred to as pixel coordinates. See [Gon04]. 3.1.2. Images as Matrices. The coordinate system discussed in preceding section leads to the following representation for the digitized

$$\mathbf{f(x,y)} = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & \vdots & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

The right side of the equation is a representation of digital image. Each element of this array(matrix) is called the pixel.

Now, in MATLAB, the digital image is represented as the following matrix:

$$\mathbf{f} = \begin{bmatrix} f(1,1) & f(1,2) & \cdots & f(1,N) \\ f(2,1) & f(2,2) & \cdots & f(2,N) \\ \vdots & \vdots & \vdots & \vdots \\ f(M,1) & f(M,2) & \cdots & f(M,N) \end{bmatrix} \tag{3.1}$$

image function

where M = the number of rows and N = the number of columns Matrices in MATLAB are stored in variables with names such as A, a, RGB, real array and so on. See [Gon04].

3.1.3. Color Image Representation in MATLAB. An RGB color image is an M × N × 3 array or matrix of color pixels, where each color pixel consists of a triplet corresponding to the red, green, and blue components of an RGB image at a specific spatial location. An RGB image may be viewed as a “stack” of three grayscale images, that when fed into the red, green, and blue inputs of a color monitor, produce a color image on the screen. So from the “stack” of three images forming that RGB color image, each image is referred to as the red, green, and blue component images by convention. Now, the data class of the component images determine their range of values. If an RGB color image is of class double, meaning that all the pixel values are of type double, the range of values is [0, 1]. Likewise, the range of values is [0, 255] or [0, 65535] for RGB images of class uint8 or uint16, respectively. The number of bits used to represent the pixel values of the component images determines the bit depth of an RGB color image. See [Gon04].

Wavelet Decomposition of an Image

Color Conversion. In the process of image compression, applying the compression to the RGB components of the image would result in undesirable color changes. Thus, the image is transformed into its intensity, hue and color saturation components. The color transformation used in JPEG 2000 standard [Sko01] has been adopted. For the lossy compression, equations (3.2) and (3.3) were used in the program.

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.16875 & -0.33126 & 0.5 \\ 0.5 & -0.41869 & -0.08131 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (3.2)$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.0 & 0 & 1.402 \\ 1.0 & 0.34413 & -0.71414 \\ 1.0 & 1.772 & 0 \end{bmatrix} \begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} \quad (3.3)$$

$$\begin{bmatrix} Y_r \\ V_r \\ U_r \end{bmatrix} \begin{bmatrix} \lfloor \frac{R+2G+B}{4} \rfloor \\ R-G \\ B-G \end{bmatrix} \quad (3.4)$$

$$\begin{bmatrix} G \\ R \\ Br \end{bmatrix} = \begin{bmatrix} Y_r - \lfloor \frac{U_r+V_r}{4} \rfloor \\ V_r + G \\ U_r + G \end{bmatrix} \quad (3.5)$$

Here, Y is the luminance and U and V are chrominance values (light intensity and color intensity), the subscript r stands for reversible. The advantage of this color system is that the human perception for the Y component is substantially more sensitive than for fluctuations in the U or V components. This can practically be used to transform U and V components are ansfered less. These components it reduces the data set of these two components to 1/4 of the original amount to be worth transferring [Han00], [Sko01].

Fig 2: Wavelet Decomposition of an Image Component. The image has been modified: the average detail has been lightened and the horizontal, vertical and diagonal details are shown as negative images with a reversal of white and black, because of contraints of the printing process.

Implementation of the Program. The program was implemented using MATLAB with various subroutines that enables the wavelet transformation, image compression and threshold computation from the Wavelet Toolkit

4. Conclusion

Wavelet compression did show remarkable performance especially with smaller threshold value; it was not differentiable in between the original image and the compressed image for some cases. However, more improvements can still be made. As it is mentioned in [Sko01] there is more room for improvement by adding more stages to the compression such as quantization, entropy encoding, etc. Also, we have not covered all the wavelets that is out there, that it cannot be decided as to which one performs the best image compression.

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