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A fixed point theorem for Junck-Mann iteration

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Abstract

In this paper, we establish the continuity of the mapping in pathak fixed point theorem for normed space useing Junck Mann.

Keywords: Mann iteration, I – quasi- non-expansive map

1. Introduction

We remark that the class of quasi- non-expansive maps properly includes the class of non-expansive maps with $F(T) \neq \Phi$. Gosh and Debnath studied the convergence of iterates of the family of non-expansive mapping in a uniformly convex Banach space. Rhoades and Temir established the weak convergence of the sequence of the Mann iterates to a common fixed point of T and I by considering the map T to be I- non-expansive.

Recently kizilton c and Ozdemir established the week convergence of the sequence of modified Ishikawa iterates to a common fixed point of T and I. kuman, Kumethong and Jewwaiworn also established the week convergence for an I- non-expansive mapping in Banach space. Our aim is to establish the weak convergence of the sequence of Mann iteration to a common fixed point of two maps T and I.

The Mann iteration scheme [1], for n= 0, 1, 2 …………………. and $\alpha_n \in [01]$ is defined as

$$X_{n+1} = (1-\alpha_n)\, x_n + \alpha_n T x_n \qquad (1)$$

For (i) $\alpha_0 = 1$, (ii) $0<\alpha_n<1$ for $n>0$, (iii) $\lim \alpha_n = h > 0$, these iterative schemes are developed by taking two mapping $S, T : Y \rightarrow x$ where $T(Y) \subseteq S(Y)$ and $x_0 \in Y$. Singh et at [1] discuss the following iterative procedure.

$$Sx_{n+1} = f(T, x_n),\ n = 0, 1, \ldots \qquad (2)$$

It is called Junek iterative procedures [2]. It $f(T, x_n)$ in [2] is replaced by Tx_n

$$(1-\alpha_n)\, Sx_{n+1} + \alpha_n Tx_n \qquad (3)$$

it becomes Junck Picard and Junck Mann iteration.

Nampally and Singh extended some fixed point theorems of Rhoades for a mapping T satisfying certain contractive conditions, if the sequence of Mann iterates converges to a fixed point of T. Let T be a self-mapping of normed space N.We recall that the G-iterative process associated with T is defined in the following manner,

Let X_0 be in N and set,

$$X_{n+1} = (\mu_n-\lambda_n)\, X_n + \lambda_n T X_n + (1-\mu_n)\, T X_{n-1} \qquad (4)$$

Where $I\mu_n I$, $I\lambda_n I$ satisfying (i) $\lambda_0 = \mu_0 = 1$, (ii) $0<\lambda_n<1$, $0\leq \mu_n \leq 1$, such $\mu_n > \lambda_n$, $n>0$, (iii) $\lim_{n\rightarrow\infty} \lambda_n = h>0$,(iv) $\lim_{n\rightarrow\infty} \mu_n = 1$,the G iterative process reduces to Mann iterative scheme.

2. Preliminaries

Let K be a closed convex banded subset of a uniformly concave Branch space (X, II. II) and T be a self-mapping of X. T is non-expansive on K if for all $x, y \in K$ we have.

$$IITx-TyII \leq IIx-yII \qquad (5)$$

A point of $f \in K$ is a fixed point of T if $Tf = f$. We denote the set of the fixed points of T by $f(T)$, where $f(T) = \{f \in K: T f = f\}$

A map T satisfying

II Tx-f II $\leq$ II x-f II (6)

X $\in$ K and f $\in$ F (T), is called a quasi- non-expansive mapping.

Definition 2.1

T is called I- non-expansive map on K if IITx-TyII $\leq$ IITx-Ty, for all x, y, $\in$ K T is called I- quasi non-expansive map on K if II Tx-f II $\leq$ II Tx-f II for all x, y $\in$K is a common fixed point of I and T if x=Ix==Tx=Sx.

3. Main Result

Theorem

Let X be a closed, convex and bounded subset of a normed space N, let T be a generalized contractive mapping of X with T continuous on X, and let (x_n), the sequence of Mann iterates associated with T, be the same as defined above where α_n satisfies (i),(ii)and(iii).If {x_n} converges in X, then it converges to a fixed point T.

Proof

Pathak finally asks if the continuity of T is necessary in the theorem for T to have a fixed point. In affirmative answer T is a generalized contractive mapping then T also satisfies the inequality

II Tx – Ty II $\leq$ q max {II x – y II, II x – Tx II, II x – Ty II, II y– Tx II, II y – Ty II
For all x,y in X, where 0< q< 1. (7)

Using inequality (6), it follows in exactly the same way as in pathak's proof of the theorem that
If $\lim_{n\to\infty} X n$ = z, then

II z – Tz II $\leq$ {II z – x_{n+1} II + (1-α_n) II x_n– TzII + α_n q max {II x_n – z II, II x_n – Tx_n II,
II x_n – Tz II, II Tx_n – z II, II z – Tz II} (8)

It now follows the definition of x_n in by (1),

$Tx_n = (x_{n+1} - (1-\alpha_n) S x_n / \alpha_n)$
$\lim_{n \to \infty} Tx_n = z$

On letting n tend to infinity in inequality (6) we now have

IIz – TzII $\leq$ (1 –h) IIz–TzII + hq max {0, II z – Tz II}
= (1 –h+hq) II z – Tz II
Where 1 –h+hq < 1 thus Tz = z (9)

This result also exists for Isikawa and Noor iteration. When Y=X and S = id = I is the identity operator on X.
Using equation (4) we have

$X_{n+1}=(\mu_n-\lambda_n)X_n + \lambda_n T X_n + (1-\mu_n) T X_{n-1}$
$T X_n = [X_{n+1} - (\mu_n-\lambda_n)X_n - (1-\mu_n) T X_{n-1}] / \lambda_n$ (10)

Then the process of Junck useing in equation (10), th

$X_{n+1}=(\mu_n-\lambda_n)SX_n + \lambda_n T X_n + (1-\mu_n) T X_{n-1}$
$T X_n = [X_{n+1} - (\mu_n-\lambda_n)SX_n - (1-\mu_n) T X_{n-1}] / \lambda_n$ (11)

Then in (10), (11)

$TX_n - TX_n = [X_{n+1} - (\mu_n-\lambda_n)X_n - (1-\mu_n)T X_{n-1}] / \lambda_n - [X_{n+1} - (\mu_n-\lambda_n)SX_n - (1-\mu_n)TX_{n-1}] / \lambda_n$ (12)
$0 = -(\mu_n-\lambda_n)X_n + (\mu_n-\lambda_n)SX_n$
$(\mu_n-\lambda_n)SX_n = (\mu_n-\lambda_n)X_n$
$SX_n = X_n$ (12)

4. References

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