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Modification of Junck-Mann Iteration

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Abstract
In this paper, we establish a new condition of Junck Mann iterationby useing Pathak fixed point theorem and also establish a new type of iteration with Newton-Rphson method. and Squara Root method which is also introducing converges to a fixed point theorem.

Keywords: Mann iteration, I – quasi- nonexpansive map

1. Introduction

We remark that the class of quasi- nonexpansive maps properly includes the class of nonexpansive maps with F(T) $\neq\Phi$. Gosh and Debnath studied the convergence of iterates of the family of nonexpansive mapping in a uniformly convex Banach space. Rhoades and Temir established the weak convergence of the sequence of the Mann iterates to a common fixed point of T and I by considering the map T to be I- nonexpansive.

Recently kizilton c and Ozdemir established the week convergence of the sequence of modified Ishikawa iterates to a common fixed point of T and I. kuman, Kumethong and Jewwaiworn also established the week convergence for an I-nonexpansive mapping in Banach space. Our aim is to establish the weak convergence of the sequence of Mann iteration to a common fixed point of two maps T and I (5, 6).

The Mann iteration scheme [1,4], for n= 0, 1, 2 …………………. and $\alpha_n \in [0,1]$ is defined as

$$X_{n+1} = (1-\alpha_n)\, x_n + \alpha_n T x_n \qquad (1)$$

For (i) $\alpha_{0=} 1$, (ii) $0<\alpha_n<1$ for n>0, (iii) lim α_n =h >0 ,these iterative schemes are developed by taking two mapping S, T : Y $\rightarrow$ x
where T(Y) $\subseteq$ S (Y) and $x_0 \in$ Y. Singh discuss the following iterative procedure.

$$Sx_{n+1} = f(T, x_n)\ , n = 0, 1, \dots \qquad (2)$$

It is called Junek iterative procedures [2, 3]. It f (T, x_n) in is replaced by Tx_n

$$(1-\alpha_n)\, Sx_{n+1} + \alpha_n Tx_n \qquad (3),$$

it becomes Junck Picard and Junck Mann iteration.

2. Preliminaries

Let K be a closed convex banded subset of a uniformly concave Branch space (X, II. II) and T be a self-mapping of X. T is nonexpansive on K if for all x, y $\in$ K we have.

$$\text{IITx-TyII} \leq \text{IIx-yII} \qquad (4)$$

A point of f $\in$ K is a fixed point of T if Tf =f. We denote the set of the fixed points of T by f (T), where
f (T) ={f $\in$ K : T **f** =f}

A map T satisfying

$$\text{II Tx-f II} \leq \text{II x-f II} \qquad (5)$$

X $\in$ K and f $\in$ F (T), is called a quasi- nonexpansive mapping.

Definition 2.1: T is called I- nonexpansive map on K if IITx-TyII $\leq$ IITx-Ty,for all x, y, $\in$ K T is called I- quasi nonexpansive map on K if II Tx-f II $\leq$ II Tx-f II for all x, y $\in$K is a common fixed point of I and T if x=Ix==Tx=Sx.

Definition 2.2: (Newton-Rphson method) Let x0 denote an approximate value of the desired root of the equation Tx = 0 and let h be the correction which must be applied to x0 to get the exact value of root. Then we have (x0 + h) is a root of the equation T(x) =0, so that T(x0 + h) =0

$$T(x0+h) = T(x0) + h\,T'(x0) + (h^2/2)T''(x0) + \dots\dots = 0$$

Now if h is sufficiently small, we may neglect the terms containing second and higher powers of h and get simple relation T(x0) + h T´(x0) = 0
This gives, h **=** - T(x0)/ T´(x0) ,provided T´(x0) $\neq$ 0. The improved value of the root is

$$X_{1} = X_0 + h = X_0 - T(x0)/T'(x0)$$

Successive approximations are given by X_2, X_3, X_4, X_5,…………………………… X_n+1 , where

$X_n+1=X_n-T(x0)/T'(x0)$ (6)

Definition 2.3: (Squara Root) The quantity √a can be considered as a root of the equation x^2 - a =0. From this we get the recursion formula

$X_n+1=[X_n-(a/X_n)]$ (7)

3. Main Result

Theorem: Let X be a closed, convex and bounded subset of a normed space N ,let T be a generalized contractive mapping of X with T continuous on X, and let (x_n),the sequence of Mann iterates associated with T, be the same as defined above where α_n satisfies (i),(ii)and(iii).

If {x_n} converges in X , then it converges to a fixed point T.

Proof- Pathak finally asks if the continuity of T is necessary in the theorem for T to have a fixed point. The definition of Mann(1) and Junek Mann (3) iterative scheme satisfies equation

$Tx_n=x_{n+1}-(1-\alpha_n)x_n/\alpha_n$ (8)

$Tx_n = x_{n+1} - (1-\alpha_n)S\, x_n/\alpha_n$ (9)

From equatin(8), (9)

$Tx_n - Tx_n = [x_{n+1} - (1-\alpha_n)\, x_n/\alpha_n] - [x_{n+1} - (1-\alpha_n)S\, x_n/\alpha_n]$

$(1-\alpha_n)S\, x_n = (1-\alpha_n)\, x_n$

$S\, x_n = x_n$ (10)

Using (2.2) Successive approximations are given by X_2, X_3, X_4, X_5,..................................... X_n+**1** , where

$X_n+1=X_n-T(x0)/T'(x0)$ (11)

thus

$X_n=X_n+\mathbf{1}+T(x0)/T'(x0)$ (12)

Then we use put the value of X_n in equation (10),

$S\, x_n = x_n$

$S\, x_n = X_n+\mathbf{1}+T(x0)/T'(x0)$ (13)

Thus we find a Newton-Rphson jungck iteration in special case.

Using (2.3) $X_n+1 = [X_n - (a/X_n)]$

$X_n = [X_n+1 + (a/X_n)]$

Then by (10) $S\, x_n = x_n$

$S\, x_n=[X_n+1+(a/X_n)]$ (14)

This result also exists for Isikawa and Noor iteration. when Y=X and S = id = I is the identity operator on X.

Example: The root of equation x^3 –3x–5=0.

Proof: We have $Tx = x^3 - 3x - 5$, $T'x = 3x^2 - 3$.

Also T(2)= -3 and, T(3)=13,hence root lies between 2 and 3.

Then we take $x_0 = \mathbf{3}$, by (11)

$X_n+1= X_n - T(x0)/T'(x0)$, n=0,1,2,3.,…….

$X1 = X_0 - T(x0)/T'(x0)$,

$=3-(3^2-3.3-5/3.3^2-3)=2.46$

$X_n+1= 2.46$

Then

$x_n = X_n+\mathbf{1} + T(x0)/T'(x0)$

$= X_n+\mathbf{1} + T(x0)/T'(x0)$

$=2.46 +(3^2-3.3-5/3.3^2-3)$

$=2.46 + 0.541666$

$=3.001.$

Thes we can say that $S\, x_n = x_n$, exist .

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