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# A Bayesian study on the Half-Normal distribution using No informative priors

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**Abstract**
In this paper, the distribution of absolute value of a normal random variable is deliberated under Bayesian structure. This distribution is also known as Half-normal distribution (HND). The posterior distributions of the parameter of HND are derived using non-informative priors which are uniform and Jeffreys priors. The properties of posterior distributions are discussed via simulation and real data set. The graphs of the posterior distribution are presented with real data set. Bayes estimates are obtained using the loss functions (Squared error loss function, Modified loss function, Quadratic loss function and Degroot loss function). Interval estimates are constructed through credible intervals. Bayesian hypothesis testing using both priors is also presented in this study.

**Keywords:** Non-informative priors, Uniform Prior (UP), Jeffreys Prior (JP), credible intervals (CI), Bayesian hypothesis testing, Loss Functions.

## Introduction

Bayesian statistics offers a rational theory of beliefs in context of uncertainty with the central aim of characterizing, how an individual should act in order to avoid certain kinds of undesirable behavioral inconsistencies. There is no better way to go than Bayesian analysis with noninformative priors. Berger (1985) argues that when information is not in compact form the Bayesian analysis using non-informative priors or single most suitable consideration.

In some problems the random variable of interest is the absolute value of the measurement. Thus, if we are measuring the deviation from a standard and are not interested in whether the reading is positive or negative, then the distribution of the original data is the normal, the appropriate model to use for the absolute values is a half-normal distribution. Jorge and Tom (1997) did visual inspection of a half-normal plot which is a popular procedure for interpreting data from replicated factorial experiment. Taylor (1994) has considered the method using the half-normal plots of obtaining a bad answer to problem which would have no answer at all corresponding to the situation in which the analysis of variance is not carried out, because of the amount of unfamiliar arithmetic involved or have with grapple with intractable computer packages. Wiper *et al*. (2005) have discussed approaches to Bayesian inference for the half-normal distribution and half-t distributions. Kahadawala and Malwane (2008) proposed new distribution which shows more exclusive mathematical tractability and statistical attractiveness with a flexible thicker left tail than the other leading lifetime distributions such as Weibull, Gamma, Lognormal etc. Rodrige *et al*. (2010) proposed a Beta generalized half-normal distribution to extend the half normal distribution and the generalized half-normal model by Cooray and Ananada (2008). For the first time, Gauss M. Cordeiro (2012) study the Kumaraswamy generalized half-normal distribution for modeling skewed positive data. The half-normal and generalized half-normal (Cooray and Ananda, 2008) distributions are special cases of the new model. Several of its structural properties are derived, including explicit expressions for the density function, moments generating and quantile functions, mean deviations and moments of the order statistics. In the article of Rao (2014) a group acceptance sampling plan is developed based on truncated lifetimes when the lifetime of an item follows a half normal distribution. For a given group size, the minimum number of groups and the acceptance number required are determined for specified consumer's risk and the test termination time. In the study of Meiping (2015) a new class of slash distribution is studied for analyzing nonnegative data. The distribution is defined by means of stochastic representation as a mixture of a half-normal random variable with the power of an exponential random variable. Density function and properties involving hazard function, moments and moment generating function are derived. These studies give mathematical handling to half-normal distribution but ignore the application aspect of the half-normal distribution. In this paper, the model of absolute value of a normal random variable and its likelihood function are presented and the posterior distribution using non-informative priors are derived in section 2. The section 3 presents the graphs of the posterior distribution using real data set. Section 4 explores Bayes estimates under different Loss functions. In section 5, Bayes estimates and posterior risks using real data set are given. Credible intervals and Hypothesis testing using real data set are explained in section 6. Section 7 contains simulation study where the computations involved are conducted using Mathematica and SAS packages. Section 8 consists of conclusion and some attractive remarks.

## 2. Posterior Distribution of the Parameter of Half-Normal Distribution

Let $x_1, x_2, \ldots, x_n$ be a random sample taken from Half-normal distribution with location parameter zero and unknown scale parameter $\theta$ for a random variable X is:

$$f(x;\theta) = \sqrt{\frac{2}{\pi}}\frac{1}{\theta}exp\left\{-\left(\frac{x^2}{2\theta^2}\right)\right\},\ \ \theta > 0, 0 < x < \infty \qquad 2.1$$

The likelihood function of the Half-normal distribution with unknown parameter $\theta$ is:

$$L(\theta,\mathbf{x}) = \left(\sqrt{\frac{2}{\pi}}\right)^n \frac{1}{\theta^n}exp\left\{-\left(\frac{\sum x^2}{2\theta^2}\right)\right\} \qquad 2.2$$

**2.1 Posterior Distribution Using Non-Informative Prior**

A prior distribution is non-informative if the prior is "flat" relative to the likelihood function. Thus, a prior is □(□) non-informative if it has minimal impact on the posterior distribution of□. Other names for the non-informative prior are vague, diffuse, and flat prior. Many statisticians favor non-informative priors because they appear to be more objective. Berger (1985) argues that when information is not in compact form the Bayesian analysis using non-informative priors or single most suitable consideration. However, it is unrealistic to expect that non-informative priors represent total ignorance about the parameter of interest.

**2.1.1 Posterior Distribution Using Uniform Prior**

The Uniform prior is first introduced by Laplace (1812). The standard Uniform distribution is assumed as non-informative prior for the parameter. The uniform prior for □ is

$$p(\theta) \propto 1,\ 0 < \theta < \infty \qquad 2.3$$

This is also improper prior.

Using equations (2.2) and (2.3), the posterior distribution of the parameter $\theta|\mathbf{x}$is:

$$p(\theta|\mathbf{x}) \propto p(\theta)L(\theta,x)$$

$$p(\theta|\mathbf{x}) \propto \theta^{-\left[2\left(\frac{n-1}{2}\right)+1\right]}exp\left\{-\left(\frac{\sum x^2}{2\theta^2}\right)\right\},\ 0 < \theta < \infty \qquad 2.4$$

Which is the density kernel of squared root inverted gamma (SRIG) distribution, so the posterior distribution of $\theta|\mathbf{x}$ is

$$SRIG(\alpha_u,\beta_u)\ where\ \alpha_u = \frac{n-1}{2}, and\ \beta_u = \frac{\sum x^2}{2}.$$

**2.1.2 Posterior Distribution using Jeffreys Prior**

The Jeffreys prior named after Harold Jeffreys (1946) is non-informative prior or objective or reference prior distribution which is obtained as square root of the determinant of the information matrix.

Hence the Jeffreys prior of the parameter $\theta$ is

$$p(\theta) \propto \frac{1}{\theta}, 0 < \theta < \infty$$

Using equations (2.2) and (2.5), the posterior distribution of the parameter $\theta|\mathbf{x}$ is:

$$p(\theta|\mathbf{x}) \propto \frac{2\left(\frac{\sum x^2}{2}\right)^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)}\theta^{-\left[2\left(\frac{n}{2}\right)+1\right]}exp\left\{-\left(\frac{\sum x^2}{2\theta^2}\right)\right\} \qquad 2.6$$

Which is the density kernel of squared root inverted gamma (SRIG) distribution, so the posterior distribution of $\theta|\mathbf{x}$ is

$$SRIG(\alpha_J,\beta_J)\ where\ \alpha_J = \frac{n}{2}, and\ \beta_J = \frac{\sum x^2}{2}.$$

**3. Graphs and Analysis with Real Life Data Set**

In this section, we represent the graphs of the posterior distribution using non-informative priors. We draw graphs in SAS package.

**3.1 Real Data Set**

The real data set used for analysis. From the Kahadawala *et al*. (2008), the following 50 data points represent the stress-rupture life of Kevlar 49/epoxy strands which were subjected to constant sustained pressure at the 90% stress level until all had failed, so we have exact times of failure in hours are given below (Andrews and Herzberg, 1985; Barlow *et al*., 1984).

0.01, 0.01, 0.02, 0.02, 0.02, 0.03, 0.03, 0.04, 0.05, 0.06, 0.07, 0.07, 0.08, 0.09, 0.09, 0.10, 0.10,0.11, 0.11, 0.12, 0.13, 0.18, 0.19, 0.20, 0.23, 0.24, 0.24, 0.29, 0.34, 0.35, 0.35, 0.38, 0.40, 0.42, 0.43, 0.52, 0.54, 0.56, 0.60, 0.60, 0.63, 0.65, 0.67, 0.68, 0.72, 0.72, 0.72, 0.73, 0.79, 0.79.

**3.1.1 Graphs of Posterior Distribution using Non-Informative Priors**

The graphs of posterior distribution using uniform prior with parameters $\alpha_u = 24.5, \beta_u = 3.9727,$ and Jeffreys prior with parameters$\alpha_J = 25, \beta_J = 3.9727$ are presented below in Figs 3.1 and 3.2

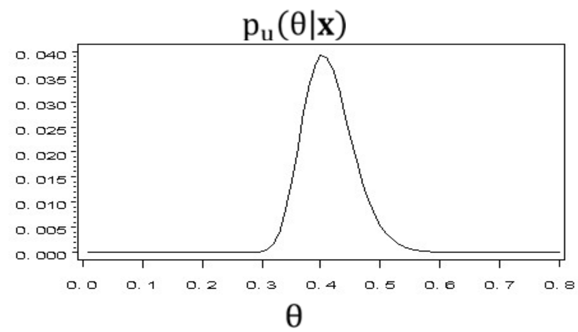


**Fig 3.1:** Graph of Posterior distribution Using Uniform Prior

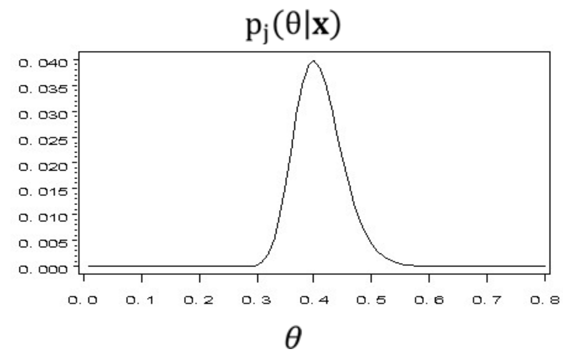


**Fig 3.2:** Graph of Posterior Distribution Using Jeffreys Prior

The graphs of posterior distributions using non-informative priors in Figs 3.1 and 3.2 are similar and slightly positively skewed.

**3.2 Properties of Posterior Distribution Using Real Data Set**

The properties of posterior distribution using real data set mentioned in 3.1 are determined which are given below

**Table 3.1:** Properties of Posterior Distribution

| n=50 | Mean | Variance | Mode | C.V |
|---|---|---|---|---|
| Uniform Prior | 1.95248 | 0.008426 | 0.39864 | 2.5226% |
| Jeffreys Prior | 1.95330 | 0.002825 | 0.39544 | 4.8093% |

From the above Table 3.1, if we compare non-informative priors, Jeffreys prior is most efficient than Uniform prior distribution, as variance is smaller using Jeffreys prior.

## 4. Bayes Estimates Under Different Loss Functions

Loss function $L(\theta,\hat{\theta})$ that describes the —loss‖ incurred by making an estimate ˆ when the true value of the parameter is $\theta$ .In Bayes statistics, loss function is a function that maps an event onto real number intuitively representing some "cost" associated with the event. The loss function itself is random quantity because it depends on the outcome of a random variable *X*. In this section, we have used four different loss functions. The details are given below:

### 4.1 Squared Error Loss Function (SELF)

The loss function: $L(\theta,\hat{\theta}) = (\theta - \hat{\theta})^2$ is called squared error loss function where $\theta$ is the parameter and $\hat{\theta}$ is an estimator. It was proposed by the Legendre (1805) and Gauss (1810) to develop least square theory.

By minimizing the risk function $\rho(\hat{\theta}) = EL(\theta,\hat{\theta})$ with respect to $\theta$, we have Bayes estimator

$$\hat{\theta} = E(\theta) \quad 4.1$$

which is the posterior mean under SELF.

The Bayes posterior risk is

$$\rho(\hat{\theta}) = E(\theta^2) - \{E(\theta)\}^2 \quad 4.2$$

Which is the posterior variance and it is the Bayes posterior risk under squared error loss function.

### 4.2 Quadratic Loss Function (QLF)

The loss function: $L(\theta,\hat{\theta}) = \left(1 - \frac{\hat{\theta}}{\theta}\right)^2$ is called quadratic loss function. Quadratic loss functions were introduced in the 1700's and 1800's, and the more recently have been advocated by Taguchi (Taguchi & Wu, 1979) and others for process optimization.

By minimizing the risk function, we have

$$\hat{\theta} = \frac{E(\theta^{-1})}{E(\theta^{-2})} \quad 4.3$$

which is the Bayes estimator under QLF .

The Bayes posterior risk is

$$\rho(\hat{\theta}) = 1 - \frac{\{E(\theta^{-1})\}^2}{E(\theta^{-2})} \quad 4.4$$

This is the Bayes posterior risk under quadratic loss function.

### 4.3 Modified Loss Function (MLF)

The loss function $L(\theta,\hat{\theta}) = \frac{(\theta-\hat{\theta})^2}{\theta}$ is called modified loss function. Modified loss function is considered as candidate when solving the problem of testing hypothesis (see, e.g., Hwang *et al*. (1992) and Robert and Casella (1994)).

By minimizing the risk function, we have

$$\hat{\theta} = \frac{1}{E(\theta^{-1})} \quad 4.5$$

which is the Bayes estimator under MLF.

The Bayes posterior risk is

$$\rho(\hat{\theta}) = E(\theta) - \frac{1}{E(\theta^{-1})} \quad 4.6$$

This is the Bayes posterior risk under modified loss function.

### 4.4 Degroot Loss Function (DLF)

The loss function $L(\theta,\hat{\theta}) = \left(\frac{\theta-\hat{\theta}}{\hat{\theta}}\right)^2$ is called Degroot loss function. In 1970, Degroot discussed different types of loss functions and obtained the Bayes Estimates under these loss functions. By minimizing the risk function, we have

$$\hat{\theta} = \frac{E(\theta^2)}{E(\theta)} \quad 4.7$$

which is the Bayes estimator under DLF.

The Bayes posterior risk is

$$\rho(\hat{\theta}) = \frac{Var(\theta)}{E(\theta)} \quad 4.8$$

This is the Bayes posterior risk under Degroot loss function.

The expressions of Bayes estimators and posterior risks using Uniform and Jeffreys priors are given in Tables 4.1 and 4.2

**Table 4.1:** Bayes Estimators and Posterior Risks Assuming Uniform Prior

| Loss Functions | Bayes Estimators | Posterior Risks |
|---|---|---|
| SELF | $\hat{\theta} = \sqrt{\frac{\sum x^2\,\Gamma(\frac{n}{2}-1)}{2\,\Gamma(\frac{n-1}{2})}}$ | $\rho(\hat{\theta}) = \frac{\sum x^2}{n-3} - \left(\sqrt{\frac{\sum x^2\,\Gamma(\frac{n}{2}-1)}{2\,\Gamma(\frac{n-1}{2})}}\right)^2$ |
| QLF | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}}\,\frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n+1}{2})}$ | $\rho(\hat{\theta}) = 1 - \left[\left(\frac{1}{\frac{n-1}{2}}\right)\left(\frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}\right)^2\right]$ |
| MLF | | |

| | | |
|---|---|---|
| | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$ | [illegible] |
| DLF | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n-3}{2}\right)}{\Gamma\left(\frac{n-2}{2}\right)}$ | [illegible] |

**Table 4.2:** Bayes Esti mators and Posterior Risks As suming Jeffreys Prior

| Loss Functions | Bayes Estimators | Posterior Risks |
|---|---|---|
| SELF | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$ | $\rho(\hat{\theta}) = \frac{\sum x^2}{n-1} - \left(\sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\right)^2$ |
| QLF | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)}$ | $\rho(\hat{\theta}) = 1 - \left[\left(\frac{1}{\frac{n}{2}}\right)\left(\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\right)^2\right]$ |
| MLF | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)}$ | [illegible] |
| DLF | $\hat{\theta} = \sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$ | $\rho(\hat{\theta}) = \frac{\frac{\sum x^2}{n-1} - \left[\left(\sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n}{2}-1\right)}{\Gamma\left(\frac{n}{2}\right)}\right)^2\right]}{\sqrt{\frac{\sum x^2}{2}} \frac{\Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}}$ |

We have simulated values of Bayes estimators and posterior risks given in Appendix under different loss functions. If we compare Bayes estimators using non-informative priors, we can see the simulated values are more close to the true parameter, as we increase our sample sizes under different loss functions. For SELF we have precise results, we conclude that Bayes estimates using Jeffreys prior are better than uniform prior.

## 5. Bayes Estimation and Posterior Risk Using Real Data Set

By using the above Loss functions the Bayes estimates and Posterior Risks of the parameter through non-informative priors i.e. Uniform and Jeffreys priors are as follow where posterior risk are in parentheses.

**Table 5.1:** Bayes Estimates and Posterior Risk under Different loss Functions

| n=50 | Prior Distributions | |
|---|---|---|
| Loss Functions | UP | JP |
| SELF | 1.91577 (0.00179) | 1.95696 (0.00172) |
| QLF | 1.95774 (0.01015) | 1.95847 (0.00996) |
| MLF | 1.95698 (0.00424) | 2.03685 (0.00412) |
| DLF | 1.95530 (0.00047) | 1.95616 (0.00045) |

If we compare non-informative priors we observe that posterior risk using Jeffreys prior is less than uniform prior hence Jeffreys prior giving more efficient results. We observe that MLF performance in terms of posterior risk is better than other loss functions.

## 6. Bayesian Point and Interval Estimates Using Real Data Sets

In this section, we obtained Bayesian point and interval estimates. The Bayesian analog of a classical confidence is called a credible set. The details about credible sets see Saleem and Aslam (2009), Lynn *et al*. (2003). Saleem and Raza (2011), and Yongsheng Zhu (2007).The Bayesian credible intervals are obtained by using the posterior distribution of the respective parameter of interest.

### 6.1 Credible Intervals

A credible interval or Bayesian confidence interval is an interval, in which domain of a posterior probability distribution used for interval estimation. Credible intervals are not unique on a posterior distribution.

The credible intervals are constructed as:

$$1-\alpha = p\left\{\chi^2_{\left(1-\frac{\alpha}{2},2(\alpha)\right)} < \frac{2(\beta)}{A} < \chi^2_{\left(\frac{\alpha}{2},2(\alpha)\right)}\right\}$$

and we have,

$$[C_L^{(\theta)}, C_U^{(\theta)}] = \left[\sqrt{\frac{2(\beta)}{\chi^2_{\left(1-\frac{\alpha}{2}, 2(\alpha)\right)}}}, \sqrt{\frac{2(\beta)}{\chi^2_{\left(\frac{\alpha}{2}, 2(\alpha)\right)}}}\right] \quad 6.1$$

Thus $(C_L^{(\theta)} < \theta < C_U^{(\theta)})$ is the (1□□) 100% credible interval where ‗α' and ‗β' are the respective parameters of posterior distribution.

The Credible intervals for real data set by using equation (6.1) are

**Table 6.1:** Credible Intervals using Non-Informative Priors

| Prior Distributions | 90% Credible Interval | 95% Credible Interval | 99% Credible Interval |
|---|---|---|---|
| Uniform Prior | (0.3461,0.4840) | (0.3335,0.4955) | (0.3187,0.5400) |
| Jeffreys Prior | (0.3431,0.4781) | (0.3336,0.4965) | (0.3162,0.5328) |

In comparison, we can observe that 90% credible intervals are narrower than 99% and 95%.When we compare non-informative priors credible intervals under Jeffreys prior are shorter than uniform prior.

### 6.2 Bayesian Hypothesis Testing

Hypothesis testing has been subject to polemic since its early formulation by the Neyman and Pearson in the 1930s. It is more difficult to carry out a point null hypothesis test in a Bayesian analysis. Bayesian model comparison is method of selection of based on Bayes factors.

Jeffreys (1961) gives the following typology for comparing $H_a \; vs \; H_b$ where $H_a$ is used for Null Hypothesis and $H_b$ is used for Alternative Hypothesis.

$B > 1$ $H_a$ Is Supported

$10^{-\frac{1}{2}} \leq B \leq 1$ Minimal evidence against $H_a$

$10^{-1} \leq B \leq 10^{-\frac{1}{2}}$ Substantial evidence against $H_a$

$10^{-2} \leq B \leq 10^{-1}$ Strong evidence against $H_a$

$B < 10^{-2}$ Decisive evidence against

**Table 6.2:** Hypothesis testing using Real Data Set

| $H_a \; vs \; H_b$ | Using Uniform Prior | | | Using Jeffreys Prior | | |
|---|---|---|---|---|---|---|
| | Posterior Probability | | Bayes Factor | Posterior Probability | | Bayes Factor |
| | $p(H_a)$ | $p(H_b)$ | B.F | $p(H_a)$ | $p(H_b)$ | B.F |
| $H_a: \theta \leq 0.31$ $H_b: \theta > 0.31$ | 0.0018 | 0.9981 | 0.0018 | 0.0024 | 0.9975 | 0.0024 |
| $H_a: \theta \leq 0.37$ $H_b: \theta > 0.37$ | 0.1765 | 0.8234 | 0.2143 | 0.2032 | 0.7968 | 0.2550 |
| $H_a: \theta \leq 0.42$ $H_b: \theta > 0.42$ | 0.6342 | 0.3657 | 1.7344 | 0.6721 | 0.3278 | 2.0503 |
| $H_a: \theta \leq 0.48$ $H_b: \theta > 0.48$ | 0.9421 | 0.0578 | 16.2986 | 0.9535 | 0.0464 | 20.5419 |

The above tables 6.2 shows:

□ While considering the hypothesis

$H_a: \theta \leq 0.31$ Versus $H_b: \theta > 0.31$

As $B < 10^{-2}$, we make decisive evidence against $H_a$ using uniform and Jeffreys priors.

While considering the hypothesis

$H_a: \theta \leq 0.37$ Versus $H_b: \theta > 0.37$

Bayes factor using uniform and Jeffrey priors lies between $10^{-2} \leq B \leq 10^{-1}$. So, we conclude that there is substantial evidence against the posterior distribution under $H_a$.

While considering the hypothesis

$H_a: \theta \leq 0.42$ Versus $H_b: \theta > 0.42$

As $B > 1$ we strongly supported non-informative priors against the posterior distribution under $H_a$

While considering the hypothesis

$H_a: \theta \leq 0.48$ Versus $H_b: \theta > 0.48$

As $B > 1$ using both priors especially for Jeffreys prior, we strongly supported against the posterior distribution under $H_a$.

### 7. Properties of Posterior Distribution using Simulation Study

Simulation is the process of imitating a real phenomenon with a set of mathematical formulas. Here, we discuss some properties of posterior distribution through simulation study of parameter $\theta$ . We have done all simulation in Mathematica package.

**Table 7.1:** Properties of Posterior Distribution under Uniform Prior

| n | $\theta = 2$ | | | $\theta = 4$ | | |
|---|---|---|---|---|---|---|
| | Mean | Variance | Mode | Mean | Variance | Mode |
| 50 | 1.9756 | 1.2384 | 1.9818 | 3.9284 | 1.2565 | 3.9405 |
| 100 | 1.9998 | 1.2222 | 1.9992 | 3.9858 | 1.2396 | 3.9983 |
| 500 | 2.0015 | 1.1893 | 2.0526 | 4.0851 | 1.2285 | 4.0519 |
| 1000 | 2.0006 | 1.1823 | 2.0013 | 4.0058 | 1.2046 | 4.0006 |

**Table 7.2:** Properties of Posterior Distribution under Jeffreys Prior

| n | $\theta = 2$ | | | $\theta = 4$ | | |
|---|---|---|---|---|---|---|
| | Mean | Variance | Mode | Mean | Variance | Mode |
| 50 | 1.9939 | 1.2891 | 1.9968 | 3.9829 | 1.2240 | 3.9558 |
| 100 | 1.9988 | 1.2355 | 1.9997 | 3.9952 | 1.2128 | 3.9721 |
| 500 | 2.0209 | 1.2149 | 2.0086 | 4.0096 | 1.1932 | 4.0310 |
| 1000 | 2.0054 | 1.2021 | 2.0073 | 4.0008 | 1.1875 | 4.0097 |

From the Tables 7.1 and 7.2, it is observed that as we increase our sample sizes, our simulated values through mean tend to true values of parameter. Similarly mode is closely to the true parameter as we increase sample sizes. Jeffreys prior is more precise than uniform prior in case of comparing non-informative priors. We have also simulated values of variances, which can show as we increase the sample sizes it becomes less.

### 8. Conclusion

We have presented the Bayesian analysis of Half-normal model using non-informative (uniform and Jeffreys) priors. Initially we derive posterior distributions using noninformative priors. The SAS package is used to draw graphs of posterior distributions. The properties of posterior distribution (mean, median, mode, Variance and coefficient of variation) are discussed through simulation as well as real data set. The credible intervals for 90%, 95%, and 99% using non-informative priors are constructed and Bayes factors of different hypothesis are computed. By the comparison of results, with increasing the sample size the Bayes estimates converge to the parametric values and their risks tends to be smaller. As under non-informative priors the Bayes risks for the estimates under JP are smaller than the Bayes risks assuming of UP thus JP is more suitable prior. If we compare the Bayes risk under different loss functions, namely SELF, QLF, MLF and DLF, Thus the MLF is better loss function for estimating the parameter$\theta$.

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## Appendix

**Bayes Estimates with posterior risks under Different Loss Functions**

**Table 1:** Bayes Estimates Using Non-Informative Priors Using SELF

| Prior Distribution | Uniform Prior | | Jeffreys Prior | |
|---|---|---|---|---|
| N | $\theta = 2$ | $\theta = 4$ | $\theta = 2$ | $\theta = 4$ |
| 50 | 1.9862 | 3.9808 | 2.0953 | 3.9788 |
| 100 | 1.9955 | 3.9915 | 2.0696 | 3.9892 |
| 500 | 2.0789 | 4.0023 | 2.0101 | 4.0050 |
| 1000 | 2.0886 | 4.1085 | 2.0066 | 4.0016 |

**Table 2:** Bayes Estimates using Non-Informative Priors using QLF

| Prior Distributions | Uniform Prior | | Jeffreys Prior | |
|---|---|---|---|---|
| n | $\theta = 2$ | $\theta = 4$ | $\theta = 2$ | $\theta = 4$ |
| 50 | 1.9797 | 3.9577 | 1.9865 | 3.9506 |
| 100 | 1.9898 | 3.9961 | 1.9910 | 3.9874 |
| 500 | 2.0167 | 4.0320 | 2.0934 | 4.0689 |
| 1000 | 2.0089 | 4.0093 | 2.0078 | 4.1724 |

**Table 3:** Bayes Estimates using Non-Informative Priors using MLF

| Prior Distributions | Uniform Prior | | Jeffreys Prior | |
|---|---|---|---|---|
| n | $\theta = 2$ | $\theta = 4$ | $\theta = 2$ | $\theta = 4$ |
| 50 | 2.0362 | 3.9955 | 2.0225 | 3.9843 |
| 100 | 2.0895 | 3.9997 | 2.0707 | 3.9918 |
| 500 | 1.9964 | 4.052 | 2.0082 | 4.0019 |
| 1000 | 1.9996 | 4.0793 | 2.1058 | 4.0011 |

**Table 4** Bayes Estimates using Non-Informative Priors using DLF

| Prior Distributions | Uniform Prior | | Jeffreys Prior | |
|---|---|---|---|---|
| n | $\theta = 2$ | $\theta = 4$ | $\theta = 2$ | $\theta = 4$ |
| 50 | 1.9801 | 4.0033 | 2.0144 | 4.0515 |
| 100 | 1.9825 | 4.0551 | 2.0217 | 4.0619 |
| 500 | 2.0412 | 3.9949 | 2.0580 | 3.9917 |
| 1000 | 2.0063 | 3.9985 | 2.0967 | 3.9905 |